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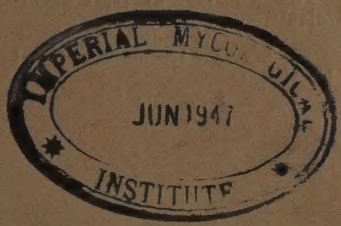

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MATHEMATICS

ON THE POSSIBILITY OF A UNIQUE METHOD IN CERTAIN PROBLEMS OF «ADDITIVE» AND «DISTRIBUTIVE» PRIME NUMBER THEORY

By U. V. LINNIK

(Communicated by I. M. Vinogradov, Member of the Academy, 28. V. 1945)

§ 1. We shall call «the distributive prime number theory» the complex of questions relating to the distribution of primes on various segments and sequences of positive integers. We shall see that the basic method of this theory is the method of Dirichlet's L-functions and the Riemann-Hadamard process of complex integration and the subsequent summation over zeros and poles of these functions.

As to the modern «additive» prime number theory, its most beautiful achievement is the Goldbach-Vinogradov theorem on three primes. The solution of the problem by I. M. Vinogradov has an unhomogeneous structure. It requires the subdivision of the segment [0,1] into the intervals of the «first» and «second» classes, closely connected with Hardy-Littlewood's «major» and «minor arcs». Whereas the «major arcs» are treated for the most part by the Riemann-Hadamard method, «minor arcs» which form the principal difficulty of the problem are treated by the method of «trigonometrical sieve» by I. M. Vinogradov. This «trigonometrical method» is now the chief method of the prime number theory.

When studying Dirichlet's theory of characters, I was led to the conclusion that many additive prime number problems may be completely treated by the Riemann-Hadamard method complemented by Carlson-Hoheisel's theorem on the density of zeros of L-functions, in various rectangles of the critical stripe.

In particular, in the present paper I expose the solution of the Goldbach-Vinogradov problem by the Riemann-Hadamard method of a straight forward summation over the zeros of L-functions.

The proof is developed by means of Hardy-Littlewood's ideas (1) with the addition of several lemmas of Carlson-Hoheisel-Titchmarsh's type on the density of the zeros of $L(w, \chi)$, and a theorem proved by myself (2). Thus, the classical L-series method, which was applied to the deduction of the law of primes in progressions, gives also Goldbach-Vinogradov's theorem. The same method gives the solution of certain problems formerly treated only by the «trigonometrical sieve method» (3), and leads also to completely new results. It may prove to be the unique method on many questions of the theory of primes.

§ 2. Let N be a large odd number which is to be represented as the sum of three primes. We shall consider the sum

$$S(N, \vartheta) = \sum_{n=1}^{\infty} \Delta(n)\, e^{-n/N} e^{-2\pi i \vartheta n}$$

We have

$$S(N,\vartheta)=\sum_{p\geqslant 2}\ln p e^{-p/N}e^{-2\pi i\vartheta p}+O(N^{1/2}\ln^2 N)$$

Hence $S(N,\vartheta)$ can be applied to the solution of Goldbach's problem, namely

$$Q(N)=e\int_0^1 S(N,\vartheta)^3 e^{2\pi iN\vartheta}d\vartheta+O(N^{3/2+\varepsilon})$$

where

$$Q(N)=\sum_{p+p'+p''=N}{}^{*}\ln p\ln p'\ln p''$$

§ 3. We put $w=\sigma+it$; $\beta\in[1/2,1]$ a real number; $\nu=\beta-1/2$. We now formulate some lemmas.

Lemma 1. For $-1/2\leqslant\sigma\leqslant 1$; $|t|\geqslant 1$ we have

$$|\Gamma(\sigma+it)|<c_1 t^{\sigma-1/2}e^{-\frac{\pi}{2}|t|} \tag{1}$$

Here Γ is Euler's function.

Lemma 2. Let χ be a primitive character (mod q); $L(w,\chi)$, the L-series belonging to it, and $T\geqslant 1$. The number of zeros of $L(w,\chi)$ in the rectangle $0\leqslant\sigma\leqslant 1$, $|t|\leqslant T$, is

$$\ll T\ln(qT) \tag{2}$$

The proofs of these lemmas are well known.

Lemma 3 (Carlson-Hoheisel-Titchmarsh). The number of zeros of the zeta-function in the rectangle $1\geqslant\sigma\geqslant\beta$, $|t|\leqslant T$, has the estimate

$$Q(\beta,T)<\min\left(c_2(\varepsilon)T^{1-\frac{2\beta-1}{2-2\beta}+\varepsilon},c_3T^{1-(\beta-1)^2}\ln^7 T\right)$$

As $\beta=1/2+\nu$, we may write

$$Q(\beta,T)<\min\left(c_2(\varepsilon)T^{1-\frac{\nu}{1-\nu}+\varepsilon},c_3T^{1-4\nu^2}\ln^7 T\right) \tag{3}$$

Proof — see ([3]) and ([4]).

Lemma 4 (generalization). If $L(w,\chi)$ be the series of Lemma 2, and $T\geqslant q^{10}$, the number of zeros of $L(w,\chi)$ in the rectangle $1\geqslant\sigma\geqslant\beta$; $|t|\leqslant T$, will be

$$Q_L(\beta,T)<\min\left\{c_2(\varepsilon)q^4T^{1-\frac{\nu}{1-\nu}+\varepsilon},c_4q^4T^{1-4\nu^2}\ln^7 T\right\} \tag{4}$$

Lemma 5. When the conditions of Lemma 4 are satisfied, we have also

$$Q_L(\beta,T)<c_5(\varepsilon)T^{1-\nu+\varepsilon} \tag{5}$$

These two lemmas of which Lemma 5 is especially important, are proved by me in a paper on «Several lemmas on the density of the zeros of L-series» (in preparation for press).

Lemma 6 is especially important. It is a consequence of the results of my papers ([2]) and ([5]) relating to the deduction of certain estimates of I. M. Vinogradov's type from the theorems on the density of zeros of L-series.

Lemma 6. There is a constant $\gamma_0 \in (0, 1)$ such that if we fix any $\lambda > 0$ and if q is any integer satisfying the conditions $\exp((\ln N)^\lambda) \leqslant q \leqslant N^{\gamma_0}$, then for $\vartheta = a/q + M/N$; $|M| < q^{100}$ we have

$$\left| \sum_{n=1}^{\infty} \Lambda(n) e^{-n/N} e^{-2\pi i \vartheta n} \right| \ll \frac{N}{q^{0.4}} \tag{6}$$

Lemma 7 (Page-Titchmarsh (6)). If σ_1 is the greatest real zero of L-functions (mod q) and $(q, l) = 1$, then

$$\pi(m, q, l) = \frac{1}{\varphi(q)} \int_2^m \frac{du}{\ln u} + O(me^{-A\sqrt{\ln m}}) + O\left(\frac{m^{\sigma_1}}{\varphi(q) \ln m}\right) \tag{7}$$

Lemma 8 (Page (6)). If $\xi \geqslant 2$, then there is at most one L-series with a primitive character (mod q), for $q \leqslant \xi$, which has a real zero in $\sigma > 1 - c_4/\ln \xi$.

§ 4. We choose $\gamma_1 = \min\{\gamma_0, 1/10^{10}\}$; $\tau_1 = N^{\gamma_1}$ (γ_0 being the number of Lemma 6). Let $H = \exp((\ln N)^{1/3})$; $H_1 = H^3$. Any $\vartheta \in [0, 1]$ can be developed into a continuous fraction

$$\vartheta = \frac{a}{q} + \alpha; \quad |\alpha| \leqslant \frac{1}{q\tau_1}; \quad q \leqslant \tau_1 \tag{8}$$

The set $\mathfrak{M}$ of numbers ϑ for which $q \leqslant H$; $|\alpha| \leqslant H_1/N = H^3/N$ is the set of numbers of the «major arcs» type. The behaviour of $S(N, \vartheta)$ on $\mathfrak{M}$, as is well known, may be studied in a classical way by means of Lemmas 7 and 8. All other numbers will form the set $\mathfrak{m}$. Let $\vartheta \in \mathfrak{m}$, $\vartheta = a/q + \alpha$. If $|\alpha| \leqslant q^{10}/N$, we have (6). Such a set of numbers ϑ will be denoted by $\mathfrak{m}_1$. For the remaining $\vartheta \in \mathfrak{m}_2$ we shall have

$$\vartheta = \frac{a}{q} + \alpha; \quad \frac{1}{q\tau_1} \geqslant |\alpha| \geqslant \frac{q^{100}}{N} \tag{9}$$

§ 5. Let χ be a primitive character (mod q) with q satisfying (8), $E(\chi) = 1$ or 0, according as χ is or is not principal, and ρ runs over the critical zeros of $L(w, \chi)$. If x is a number with $\operatorname{Re} x > 0$, we shall have, according to Littlewood (8)

$$S(N, \alpha, \chi) = \sum_{n=1}^{\infty} \chi(n) \Lambda(n) e^{-nx} = E(\chi) x^{-1} - \sum_{\rho} x^{-\rho} \Gamma(\rho) - \frac{L'}{L}(0, \chi) + \\ + \frac{1}{2\pi i} \int_{-1/2-i\infty}^{-1/2+i\infty} x^{-w} \left(-\frac{L'}{L}(w, \chi) \Gamma(w) \, dw \right)$$

provided $L(0, \chi) \neq 0$.

The case $L(0, \chi) = 0$ requires some unessential modifications.

Let $x = 1/N + 2\pi i\alpha$, where α satisfies (9). We have $|x| < 1$. For the estimation of the remaining term

$$R = \frac{1}{2\pi i} \int_{-1/2-i\infty}^{-1/2+i\infty} x^{-w} \left(-\frac{L'}{L}(w, \chi) \right) \Gamma(w) \, dw$$

we note that, on $\sigma = -1/2$, $\frac{L'}{L}(w, \chi) \ll \ln q (|t| + 2)$;

$$x^{-w} = e^{-w \ln |x| - iw \arg x}; \quad |e^{-w \ln |x|}| < 1; \quad |e^{-iw \arg x}| \leqslant e^{|t| \arg x};$$

$$\Gamma(w) \ll e^{-\frac{\pi}{2}|t|} / |t|$$

Putting $\eta = \pi/2 - \operatorname{arc}\, x = \operatorname{arctg}\, 1/2\,\pi N\alpha$, we shall have

$$R \ll \int_2^\infty e^{\left(\operatorname{arc}\, x - \frac{\pi}{2}\right)} \frac{\ln qt}{t}\, dt \ll \frac{1}{\eta \ln \frac{1}{\eta}}$$

As $N\alpha$ is large for $\vartheta \in \mathfrak{m}_2$ we have

$$\operatorname{arctg}\, 1/2\,\pi N\alpha \sim 1/2\,\pi N\alpha \text{ and } R \ll N\alpha \ln (N\alpha)$$

Furthermore, $\frac{L'}{L}(0, \chi) \ll q$; $x^{-1} \ll \alpha^{-1}$. Hence

$$S(N, \alpha, \chi) \ll \alpha^{-1} + N\alpha \ln (N\alpha) + \left| \sum_\rho x^{-\rho}\, \Gamma(\rho) \right| \tag{10}$$

§ 6. Let $\nu_0 = \gamma_1^2$. For the estimation of $\left| \sum x^{-\rho}\, \Gamma(\rho) \right|$ we divide the critical stripe σ into the stripe σ_0: $0 \leqslant \sigma \leqslant 1/2 + \nu_0 = \beta_0$, and a set of stripes σ_β: $\beta \leqslant \sigma \leqslant \beta + \frac{1}{\ln N}$; $\beta \geqslant 1/2 + \nu_0$. Let $\alpha > 0$.

Suppose that $N_0 \geqslant N\alpha$ is the number of zeros of $L(w, \chi)$ in the rectangle $0 \leqslant \sigma \leqslant \beta_0$; $|t| \leqslant N_0$; or, according to (2), $Q_L(\beta, N_0) \ll N_0 \ln (qN_0)$. Furthermore, $|x| \sim 2\pi\alpha$ and if $\rho_k = \beta_k + it_k$, we have by (1)

$$\left| \sum_{\rho_k \in \sigma\beta_0} x^{-\rho_k}\, \Gamma(\rho_k) \right| \ll \alpha^{-\beta_0} \sum_{\rho_k \in \sigma\beta_0} e^{\left(\operatorname{arc}\, x - \frac{\pi}{2}\right)|t_k|}\, |t_k|^{\beta_k - 1/2} \ll$$

$$\ll \alpha^{-\beta_0} \sum_{\rho_k \in \sigma\beta_0} e^{-|t_k|/2\pi n\alpha}\, |t_k|^{\beta_0 - 1/2} \ll \alpha^{-\beta_0} N\alpha \ln (N\alpha)\, (N\alpha)^{\nu_0} \ll \alpha^{1/2 - \nu_0} N^{2\nu_0} N$$

As by (9) $\alpha \leqslant 1/q\,\tau_1$, the required number is

$$\ll \frac{NN^{2\nu_0}}{q^{1/2-\nu_0}\,\tau_1^{1/2-\nu_0}} \ll N \frac{N^{2\nu_0}}{q^{1/2-\nu_0} N^{1/2}\,\gamma_1^{1-\nu_0}\,\gamma_1} \ll N \frac{1}{q^{1/2} N^{1/2}\,\gamma_1^{1-3\gamma_1^2}} \ll \frac{N}{q^{1/2} N^{1/4}\,\gamma_1} \tag{11}$$

§ 7. For $1/2 + \nu_0 \leqslant \beta \leqslant 0.6$, that is, $\nu_0 \leqslant \nu \leqslant 0.1$, we apply Lemma 5. It is important to note that $N_0 \geqslant N\alpha > q^{100}$. We have $Q_L(\beta, N_0) \ll c_3(\varepsilon)\, N_0^{1-\nu+\varepsilon}$. Hence

$$\left| \sum_{\rho_k \in \sigma\beta} x^{-\rho_k}\, \Gamma(\rho_k) \right| \ll \alpha^{-1/2-\nu} \sum_{\rho_k \in \sigma\beta} e^{-|t_k|/2\pi N\alpha}\, |t_k|^\nu \ll c_3(\varepsilon)\, (N\alpha)^{1-\nu+\varepsilon}\, (N\alpha)^\nu\, \alpha^{-1/2-\nu} \ll$$

$$\ll c_3(\varepsilon)\, N^{1+\varepsilon}\, \alpha^{0.4} \ll c_3(\varepsilon)\, N \frac{N^\varepsilon}{q^{0.4}\,\tau_1^{0.4}} \ll \frac{N}{q^{1/2}\,\tau_1^{0.2}} \tag{12}$$

§ 8. For $0.1 \leqslant \nu \leqslant 1/3$ we apply Lemma 4 in the form: $Q_L(\beta, N_0) \ll c_2(\varepsilon)\, q^4\, N_0^{1 - \frac{\nu}{1-\nu} + \varepsilon}$. Hence

$$\left| \sum_{\rho_k \in \sigma\beta} x^{-\rho_k}\, \Gamma(\rho_k) \right| \ll c_2(\varepsilon)\, q^4\, \alpha^{-1/2-\nu} (N\alpha)^{1 - \frac{\nu}{1-\nu} + \varepsilon} (N\alpha)^\nu \ll$$

$$\ll N^{1-\nu^2}\, \alpha^{3/2 - \frac{1}{1-\nu}} \ll N^{1-\nu^2} \ll N^{1-0.01} \tag{13}$$

In fact, $\alpha < 1$ and $\nu \leqslant 1/3$, so that $\frac{3}{2} - \frac{1}{1-\nu} \geqslant 0$. For $1/3 < \nu \leqslant 0.499$ we apply the same estimate. Here we have $\frac{3}{2} - \frac{1}{1-\nu} < 0$, and the maximum of the estimate will be attained for the best possible $\alpha \geqslant q^{100}/N$. The number in question is then

$$\ll c_3(\varepsilon)\, N^{1 - \frac{\nu^2}{1-\nu^2} + \varepsilon} \left(\frac{N}{q^{100}}\right)^{\frac{1}{1-\nu} - \frac{3}{2}} \ll c_3(\varepsilon) \frac{N^{1/2+\nu+\varepsilon}\, q^4}{q^{100\left(\frac{1}{1-\nu} - \frac{3}{2}\right)}} \ll N^{1-10^{-3}} q^4 \tag{14}$$

§ 9. For $0.499 < \nu \leqslant 1$ we apply the estimate $Q_L(\beta, N_0) \ll N_0^{1-4\nu^2} q^4 \ln^7 N_0$. This implies

$$\left| \sum_{\rho_k \in \sigma_\beta} x^{-\rho_k} \Gamma(\rho_k) \right| \ll (N\alpha)^{1-4\nu^2} (N\alpha)^{\nu} \alpha^{-1/2-\nu} q^4 \ln^7 N \ll q^4 \ln^7 N \cdot N^{1+\nu-4\nu^2} \alpha^{\frac{1}{2}-4\nu^2}$$

The estimate attains its maximal value for the best possible α; we have $\alpha > q^{100}/N$; if $q^{100} < H$, then for the considered $\vartheta \in \mathfrak{m}_2$, $\alpha > H^3/N$. Hence, $\alpha > H/N$, and the maximal estimate is

$$\ll q^4 \ln^7 N \cdot N^{1+\nu-4\nu^2} \min \left\{ \frac{N^{4\nu^2-1/2}}{H^{4\nu^2-1/2}}, \frac{N^{4\nu^2-1/2}}{q^{100(4\nu^2-1/2)}} \right\} \ll$$

$$\ll q^4 \ln^7 N \cdot N^{1+\nu-4\nu^2} \frac{N^{4\nu^2-1/2}}{H^{2\nu^2-1/4} q^{100(2\nu^2-1/4)}}.$$

As for $\nu \in [0{,}499,\ 1/2]$, $2\nu^2 - \frac{1}{4} > 0.2$, our expression is

$$\ll N^{1/2+\nu} \ln^7 N \frac{1}{q^{15} H^{0.2}} \ll \frac{N}{qH^{0.1}}$$

§ 10. If $\vartheta \in \mathfrak{m}_2$, $\vartheta = a/q + \alpha$, then gathering together the estimates (10)—(15), we find

$$\sum_{n=1}^{\infty} \chi(n) \Lambda(n) e^{-n/N} e^{-3\pi i a n} \ll |\alpha|^{-1} + N\alpha \ln(N\alpha) +$$

$$+ \frac{N}{q^{1/2} N^{1/4\nu_1}} + \frac{N}{q^{1/2} \pi_1^{0.2}} + \frac{N}{N^{0.01}} + q^4 N^{1-10^{-4}} \tag{16}$$

Hence we obviously obtain for a sufficiently small $c_5 > 0$

$$\sum_{n=1}^{\infty} \chi(n) \Lambda(n) e^{-n/N} e^{-2\pi i a n} \ll \frac{N}{q^{1/2+c_5} H^{0.1}} \tag{17}$$

§ 11. Hence we obtain for $\vartheta = a/q + \alpha$; $\vartheta \in \mathfrak{m}_2$

$$S(N, \vartheta) = \sum_{n=1}^{\infty} \Lambda(n) e^{-n/N} e^{-2\pi i \vartheta n} = \sum_{n=1}^{\infty} \Lambda(n) e^{-n/N} e^{-2\pi i \left(\frac{a}{q} + \alpha\right) n} =$$

$$= \sum_{\substack{(l,q)=1 \\ \mathrm{mod}\, q}} e^{-2\pi i \frac{a}{q} l} \sum_{\substack{n \equiv l \\ (\mathrm{mod}\, q)}} \Lambda(n) e^{-n/N} e^{-2\pi i a n} + O(q^6) =$$

$$= \frac{1}{\varphi(q)} \sum_{\chi} \left(\sum_{l} \bar{\chi}(l) e^{-2\pi i \frac{a}{q} l} \right) \sum_{n=1}^{\infty} \chi(n) \Lambda(n) e^{-n/N} e^{-2\pi i a n} + O(q^6)$$

Taking into account that $\sum_{l} \bar{\chi}(l) e^{-2\pi i \frac{a}{q} l} \ll q^{1/2+\varepsilon}$ and using (17), we find

$$S(N, \vartheta) \ll \frac{q^{1/2} \varphi(q)}{\varphi(q)} \frac{q^6}{q^{1/2+c_5} H^{0.1}} N \ll \frac{N}{H^{0.1}}$$

This estimate, together with (6), is sufficient for the solution of Goldbach's problem.

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MATHEMATICS

UNIVERSAL *K*-SPACES

By A. G. PINSKER

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In the theory of *K*-spaces (1) a particular rôle is played by the normed *K*-spaces satisfying certain special conditions (of the B_2-type) which were considered by Kantorovitch. The spaces of this kind are the simplest as regards their properties, and the most important from the point of view of applications. In particular, to these spaces belong the following spaces considered in the functional analysis: L^p, $p \geqslant 1$ (of functions summable with their p-th power), l^p, $p \geqslant 1$ (of numerical sequences $\{\xi^{(k)}\}$ for which $\sum_{k=1}^{\infty} |\xi^{(k)}|^p < +\infty$), V (of functions of bounded variation), *etc.*

Applying to such spaces the operation of extension introduced by me (2), we obtain a wider and at the same time a sufficiently simple class of *K*-spaces whose properties immediately follow from the properties of original spaces and of the operation of extension. Incidentally the spaces of this new class possess one important property consisting in that among the subspaces of the spaces of this class are to be found all possible *K*-spaces. It is just this property which leads to the notion of a universal *K*-space.

Starting from the results formulated in the present note, it is possible to solve a number of problems of the general theory of *K*-spaces and, in particular, to find necessary and sufficient conditions for the existence of continuous functionals in the *K*-spaces of the general kind.

1°. By the *K*-space (of the type K_s) is meant a linear set $X = \{x\}$ in which for certain elements is defined the relation $x > 0$, so that the following five axioms of Kantorovitch are satisfied:

I. The relations $x > 0$ and $x = 0$ are inconsistent.

II. If $x_1 > 0$ and $x_2 > 0$, then $x_1 + x_2 > 0$.

III. If $x > 0$, then there exists such an x_1 that $x_1 > 0$ and $x_1 - x > 0$.

IV. If $\lambda > 0$ and $x > 0$, $\lambda x > 0$.

V. For every bounded from above set $E \subset X$ exists the exact upper bound $\sup E$.

The *K*-spaces, normed in the sense of Banach, will be called spaces of the type *KB* with an index showing the number of the first of the five following axioms which are supposed to be satisfied in this space:

1. $\|x\| = \||x|\|$.
2. If $|x_1| < |x_2|$, then $\|x_1\| < \|x_2\|$.
3. If $x_n \to 0$ monotonously, then $\|x_n\| \to 0$.
4. If $x_n \to +\infty$ monotonously, then $\|x_n\| \to +\infty$.
5. If $x_1, x_2 \geqslant 0$, then $\|x_1 + x_2\| = \|x_1\| + \|x_2\|$.

(The spaces of the type KB_4 have been considered by Kantorovitch and were denoted by him as spaces of the type B_3).

In the spaces of the type KB_4 the convergence by the norm coincides with the topological convergence, and, consequently, for such spaces the class of B-linear functionals (continuous in the sense of convergence by the norm) coincides with the class of K-linear functionals (continuous in the sense of topological convergence).

Two K-spaces are said to be isomorphic, if is it possible to establish between their elements a one-to-one linear correspondence preserving the relations of order; if the linear correspondence preserves the norm, then the spaces are called isometric.

A subspace X_1 of a K-space X is said to be normal (is normally contained in X), if from $x \in X_1$ and $|x'| < x$ $(x' \in X)$ follows $x' \in X_1$.

For every K-space X it is possible to define its extension $\tilde{X}$ such that

1. X is normally contained in $\tilde{X}$.
2. $[\tilde{\tilde{X}}] = \tilde{X}$.
3. $\tilde{X}$ is a space with unity.
4. Whatever be the set $E = \{x_\xi\}$ of pairwise disjunctive elements from $\tilde{X}$ (*i. e.* such that $\inf(|x_{\xi'}|, |x_{\xi''}|) = 0$ for $\xi' \neq \xi''$ $\sup E < +\infty$.

Definition 1. Let $\{X_\xi\}$ be an arbitrary set of K-spaces. We define the disjunctive sum $X = \sum_\xi X_\xi$ of these spaces in the following way: as the elements of X we shall consider all possible aggregates $x = \{x_\xi\}$ where $x_\xi \in X_\xi$; if λ is a real number, $x' = \{x'_\xi\}$ and $x'' = \{x''_\xi\}$, then we put $\lambda x = \{\lambda x_\xi\}$, $x' + x'' = \{x'_\xi + x''_\xi\}$; we shall say that $x > 0$, if all $x_\xi \geqslant 0$ and not all of them are equal to zero.

Theorem 1. *The disjunctive sum of K-spaces $X = \sum_\xi X_\xi$ is a K-space and*

a) *the set of elements $X'_\xi = \{x\}$ where $x = \{x_\eta\}$ and $x_\eta = \begin{cases} x_\xi & \text{for } \eta = \xi \\ 0 & \text{for } \eta \neq \xi \end{cases}$ forms a normal subspace of X isomorphic to X_ξ, $X'_\xi = X_\xi$;*

b) *every two subspaces X'_ξ and X'_η are disjunctive in the sense that every two elements $x' \in X'_\xi$ and $x'' \in X'_\eta$ are disjunctive;*

c) *if $x_n \in X'_\xi$ and $x_n \to x$ in X, then $x_n \to x$ in X'_ξ;*

We shall also need the following

Definition 2. Let $\{X_\xi\}$ be any set of spaces of the type KB_5; we define the normed disjunctive sum of these spaces $X = \sum X_\xi$ by taking for its $x \in X$ all possible aggregates of elements from X_ξ, $x = \{x_\xi\}$, $x_\xi \in X_\xi$ satisfying the following conditions; a) only a finite or a denumerable set of the x_ξ is different from zero, and b) $\sum_\xi \| x_\xi \| < +\infty$.

The operations of addition, multiplication by a real number and the semi-ordering we define as in the general case; we norm X by putting $\| x \| = \sum_\xi \| x_\xi \|$.

Theorem 2. *A normed disjunctive sum $X = \sum_\xi X_\xi$ of spaces of the type KB_5 is a space of the type KB_5, and to every X_ξ corresponds a normal subspace $X'_\xi \subset X$ isomorphic and isometric to X_ξ; the subspaces X'_ξ and X'_η $(\xi \neq \eta)$ are disjunctive.*

Of great importance is the following theorem, which is in a certain sense inverse to theorem 2.

Theorem 3. *Every space X of the type KB_5 may be represented in the form of a normed disjunctive sum of spaces X_ξ of the type KB_5 with unity.*

Theorem 4. *The extension of a disjunctive (in particular, normed) sum of K-spaces is isomorphic to the disjunctive sum of the extensions of the summands, i. e. if* $X=\sum_\xi X_\xi$, *then* $\tilde{X}=\sum_\xi \tilde{X}_\xi$.

Theorem 5. *If X is a space of the type* KB_σ, *then there exists a space* X_1 *of the type* KB_σ *such that X is normally contained in* X_1 *and the extension of X is isomorphic to the extension of* X_1, $\tilde{X}=\tilde{X}_1$.

2°. Let X be an arbitraty K-space with unity. Consider in X a normal subspace $\overline{X}\subset X$ formed by the «bounded» elements of X, *i. e.* elements $x\in X$ for which there exists a certain constant λ such that

$$|x|\leqslant\lambda\cdot 1 \qquad (*)$$

$\overline{X}$ may be normed in the sense of Banach by putting $\|x\|$ equal to the smallest number λ satisfying the inequality (*). It is easily seen that in $\overline{X}$ $\|x\|=\|\,|(x)|\,\|$, and that if $|x_1|\leqslant|x_2|$, then $\|x_1\|\leqslant\|x_2\|$. Further, $\overline{X}$ is a Banach space and the relation $\|x_n-x\|\to 0$ implies $x_n\to x(0)$ (the inverse is, in general, not true).

Theorem 6. *The space of bounded elements generated by the extension of the disjunctive sum of K-spaces* X_ξ *is isomorphic to the disjunctive sum of spaces of bounded elements generated by each summand, i. e. in our denotations*

$$\overline{\sum_\xi \overline{X}_\xi}=\sum_\xi \overline{X}_\xi$$

In $\overline{X}$ exist B-linear functionals not identically equal to zero (the existence of non-trivial K-linear functionals will be proved in the following note). The space of B-linear functionals in $\overline{X}$ — the so-called adjoined space — shall be denoted by the symbol $\overline{X}^*=\{f\}$. $\overline{X}^*$ is a K-space and at the same time a Banach space, in which $|f\|=|f|\cdot 1$ (1 being the unity of the space X). Moreover, we have the following

Theorem 7. *The space* $\overline{X}^*$ *adjoined to the space of bounded elements of an arbitrary K-space with unity* $\overline{X}$ *is a space of the type* KB_σ.

Theorem 8. *The space* X^* *adjoined to the space X of the type* KB_σ *with unity is isomorphic and isometric to the space* $\overline{X}$ *of bounded elements of X,* $X^*=\overline{X}$. *For the unity of the space* X^* *may be taken the essentially positive linear functional* $f(x)=\|x_+\|-\|x_-\|$ $(x\in X,\ x=x_+-x_-)$.

Theorem 9. *The space* X^* *adjoined to the space X of the type* KB_σ *is isomorphic and isometric to the space of bounded elements generated by the extension of X,* $X^*=\tilde{X}$.

Theorem 10. *Let X be an arbitrary K-space with unity;* $\overline{X}$, *the normal subspace of bounded elements of X;* $\overline{X}^*$, *the space adjoined to it;* $\overline{X}^{**}$, *the space adjoined to* X^*. *Then* $\overline{X}$ *is isomorphic to a certain subspace of* $\overline{X}^{**}$, $\overline{X}\subset\overline{X}^{**}$.

In fact, for every fixed $x\in\overline{X}$ we may define a linear functional in $\overline{X}^*$ (which is, consequently, an element of $\overline{X}^{**}$) by putting $x(f)=f(x)$ $(f\in\overline{X}^*)$. The aggregate of all such functionals forms the subspace of $\overline{X}^{**}$ isomorphic to X.

3°. We proceed now to the final result of the present note.

Definition 3. The extension of every space of the type KB_σ shall be called the universal K-space.

The term «universal space» is justified by the following

Theorem 11. *Whatever be the K-space X, there exists a universal K-space U such that X is isomorphic to a part of U.*

It is evident that there exists no universal space containing all possible K-spaces (of arbitrary power), yet we may establish the following proposition: for all K-spaces whose power does not exceed a given power, there exists a common universal K-space.

In the case of discrete K-spaces this follows from the following theorem which is easily proved:

Theorem 12. *All discrete spaces of a given power are normally contained in the K-space of real functions defined on a certain abstract set.*

In the case of continuous spaces the problem remains open, and its solution must be of great importance for the general theory of K-spaces.

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MATHÉMATIQUES

SUR LA PRESQUE PÉRIODICITÉ DES SOLUTIONS DE L'ÉQUATION DES ONDES. III

Par S. L. SOBOLEFF, de l'Académie

Dans les deux Notes précédentes [1, 2] nous avons examiné les solutions de l'équation des ondes qui sont assez régulières, et nous avons établi leur presque périodicité en un sens généralisé. Maintenant nous considérons le cas général. Considérons l'équation

$$\square u \equiv \frac{\partial^2 u}{\partial x_2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} - \frac{\partial^2 u}{\partial t^2} = 0 \tag{1}$$

dans le domaine Ω des variables x, y, z, limité par une surface S dont les rayons de courbure sont continus. Nous considérons l'équation (1) avec la condition limite

$$u/_S = 0 \tag{2}$$

Introduisons la conception de solution généralisée du problème d'une manière analogue à celle que nous avons suivie pour le problème de Cauchy [3,4].

Considérons l'espace fonctionnel E des fonctions $\varphi(x, y, z, t)$, continues dans Ω jusqu'à S et qui sont non nulles seulement pour un intervale fini des valeurs de t: $t_1 \leqslant t \leqslant t_2$.

Une suite φ_n sera dite convergente (convergence forte) vers la fonction φ, si 1) φ_n converge vers φ uniformément, 2) il existe un intervalle $T_1 \leqslant t \leqslant T_2$ hors duquel toutes les fonctions φ_n s'annulent simultanément.

Soit E^* l'espace des fonctionnelles linéaires dans E. Nous désignons par ρ les éléments de E^*, et par (ρ, φ) la valeur de la fonctionnelle ρ pour la fonction φ.

Une fonctionnelle $\rho_1 \in E^*$ sera dite la solution généralisée du problème des ondes, ou bien la solution généralisée de l'équation (1) dans la condition (2), si chaque fonction φ, continue ainsi que ses dérivées de deux premiers ordres dans le domaine fermé $\Omega + S$ et telle que

$$\varphi/_S = 0 \tag{3}$$

vérifie l'égalité

$$(\rho_1, \square\varphi) = 0 \tag{4}$$

Il peut arriver que la fonctionnelle (ρ, φ) est représentable dans la forme

$$(\rho, \varphi) = \int_{-\infty}^{+\infty} \int \ldots \int_{\Omega} \rho(x, y, z, t)\ \varphi(x, y, z, t)\, d\Omega\, dt \tag{5}$$

où ρ est une fonction sommable que nous appellerons noyau de la fonctionnelle.

Si la fonctionnelle ρ_1 est une solution généralisée du problème et si elle possède le noyau $\rho_1(x, y, z, t)$, nous dirons que ce noyau est à son tour une solution généralisée du problème. On peut démontrer qu'une fonction $\rho_1(x, y, z, t)$, qui est une solution généralisée du problème et qui est continue dans le domaine fermé $\Omega + S$, ainsi que ses dérivées des deux premiers ordres, sera non seulement une solution généralisée, mais en même temps une solution ordinaire de l'équation (1) satisfaisant à la condition (2) (comparez avec (3) et (4)).

Pour l'équation des ondes, même dans des espaces fonctionnels plus généraux que E^*, il existe toujours des solutions du problème qui ne sont pas représentables dans une forme analogue à (5), contrairement à ce qui a lieu pour les équations du type elliptique.

Considérons la fonction

$$\omega(\xi) = \begin{cases} 1 & 0 \leqslant \xi \leqslant \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} \operatorname{th} \dfrac{\xi - \frac{3}{4}}{\left(\xi - \frac{1}{2}\right)(1-\xi)} & \frac{1}{2} \leqslant, \xi \leqslant 1 \\ 0 & 1 \leqslant \xi \end{cases} \tag{6}$$

La fonction $\omega(\xi)$ a toutes ses dérivées continues dans l'intervalle $0 \leqslant \xi \leqslant 1$.

Soit

$$2 \int_0^1 \omega(\xi)\, d\xi = \varkappa \tag{7}$$

La fonctionnelle ρ_h, définie par l'égalité

$$(\rho_h, \varphi) = \left(\rho, \frac{1}{\varkappa h} \int_{t-h}^{t+h} \omega\left(\left|\frac{\xi - t}{h}\right|\right) \varphi(x, y, z, \xi)\, d\xi\right) \tag{8}$$

sera nommée la valeur moyenne de la fonctionnelle ρ avec le poids ω. Si (ρ, φ) est représentable dans la forme (5), la fonctionnelle (ρ_h^*, φ) sera à son tour représentable dans cette forme, et la relation suivante a lieu

$$\rho_h(x, y, z, t) = \frac{1}{\varkappa h} \int_{t-h}^{t+h} \omega\left(\left|\frac{\xi - t}{h}\right|\right) \rho(x, y, z, \xi)\, d\xi \tag{9}$$

Nous démontrons le théorème fondamental suivant.

Théorème. *Pour chaque fonctionnelle ρ, qui est une solution généralisée du problème, la valeur moyenne avec le poids ω est à son tour une fonctionnelle (ρ_h, φ) formant une solution généralisée du problème et représentable dans la forme* (5). *Le noyau de cette fonctionnelle admet des dérivées continues de tous les ordres à l'intérieur de Ω et des dérivées continues du second ordre dans le domain fermé.*

Donnons les traits principaux de la démonstration de ce théorème.

Soit R le minimum des rayons de courbure de la surface S. Construisons une famille de fonctions $\varphi_{h,\eta}(x, y, z, t;\ x_0, y_0, z_0, t_0)$, $\eta \leqslant R$, $\eta \leqslant {}^1/_2 h$, dépendant de deux paramètres h, η et de quatre variables x_0, y_0, z_0, t_0. Soit

$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \tag{10}$$

$$\varphi_{h,\eta}^{(1)} = \frac{1}{4\pi h \varkappa} \frac{1}{r} \omega\left(\frac{|t - t_0| + r}{h}\right) \omega\left(\frac{r}{\eta}\right)$$

Si la distance δ du point M_0 aux coordonnées x_0, y_0, z_0 à la surface S est supérieure à η, nous posons

$$\varphi_{h,\eta} = \varphi^{(1)}_{h,\eta} \tag{11}$$

Si $\delta < \eta$, nous construisons le point M_1, situé sur le prolongement de la normale à la surface S, qui passe par le point M_0 dont la distance à S est égale à δ, et qui se trouve de l'autre côté de S. Soit r_1 la distance du point variable M aux coordonnées x, y, z du point M_1. Posons

$$\varphi^{(2)}_{h,\eta} = -\frac{1}{4\pi h x}\,\frac{1}{r_1}\,\omega\left(\frac{|t-t_0|+r_1}{h}\right)\omega\left(\frac{r_1}{\eta}\right) \tag{12}$$

Définissons encore la fonction $\varphi^{(3)}_{h,\eta}$ par les conditions

$$\Delta\varphi^{(3)}_{h,\eta} = \frac{\partial^2\varphi^{(3)}_{h,\eta}}{\partial x^2} + \frac{\partial^2\varphi^{(3)}_{h,\eta}}{\partial y^2} + \frac{\partial^2\varphi^{(3)}_{h,\eta}}{\partial z^2} = 0 \tag{13}$$

$$\varphi^{(3)}_{h,\eta}|_S = -\varphi^{(1)}_{h,\eta}|_S - \varphi^{(2)}_{h,\eta}|_S \tag{14}$$

Définissons maintenant la fonction $\varphi_{h,\eta}$ par la formule

$$\varphi_{h,\eta} = \varphi^{(1)}_{h,\eta} + \varphi^{(2)}_{h,\eta} + \varphi^{(3)}_{h,\eta} \tag{15}$$

La fonction

$$\varphi_{h,\eta_1} - \varphi_{h,\eta_2} = \psi$$

aura des dérivées continues du second ordre jusqu'au contour et sera nulle sur S. Par conséquent

$$(\rho,\ \Box\psi) = (\rho_1,\ \Box\varphi_{h,\eta_1} - \Box\varphi_{h,\eta_2}) = 0 \tag{16}$$

Il s'ensuit que la quantité $(\rho_1,\ \Box\varphi_{h,\eta})$ ne dépend pas de η. Posons

$$(\rho_1,\ \Box\varphi_{h,\eta}) = \rho'_h(x_0, y_0, z_0, t_0) \tag{17}$$

Du fait que la fonctionnelle ρ_1 est linéaire on obtient l'identité:

$$\int_{-\infty}^{+\infty}\int\cdots\int_{\Omega}\rho'_h(x_0, y_0, z_0, t_0)\,\varphi(x_0, y_0, z_0, t_0)\,d\Omega_0\,dt_0 =$$

$$= \left(\rho_1 \int_{-\infty}^{+\infty}\int\cdots\int_{\Omega}\Box\varphi_{h,\eta}\varphi(x_0, y_0, z_0, t_0)\,d\Omega\,dt_0\right) \tag{18}$$

En tenant compte de ce que pour $\eta < \delta$

$$\Box\varphi_{h,\eta} = \Box_0\varphi_{h,\eta} \tag{19}$$

et en intégrant par parties l'intégrale dans la formule (18) on peut établir la relation

$$\int_{-\infty}^{+\infty}\int\cdots\int_{\Omega}\rho'_h(x_0, y_0, z_0, t_0)\,\varphi(x_0, y_0, z_0, t_0)\,d\Omega_0\,dt_0 =$$

$$= \left(\rho_1 \frac{1}{2h}\int_{t-h}^{t+h}\varphi(x, y, z, \xi)\,\omega\left(\left|\frac{\xi-t}{h}\right|\right)d\xi\right) \tag{20}$$

donc, ρ'_h est le noyau de la fonctionnelle ρ_{1h}. De la formule (17) il suit immédiatement que ρ'_h sera indéfiniment dérivable.

De la définition de $\varphi_{h,\eta}$ il résulte que les fonctions $\frac{\partial^k}{\partial t^k}\Box\varphi_{h,\eta}$ sont bornées par un nombre $K(k, h, \eta)$ ne dépendant pas de la position du point M_0, et que ses fonctions tendent vers zéro en chaque point M_0 quand M_0 tend vers S.

En vertu des propriétés de la convergence faible nous pouvons conclure que

$$\lim_{M_0 \to S} \frac{\partial^k \rho_h}{\partial t_0^k} = \lim_{M_0 \to S} \left(\rho, (-1)^k \frac{\partial^k \Box \varphi_{h,\eta}}{\partial t^k} \right) = 0 \tag{21}$$

Toutes les fonctions ρ_h sont des solutions de l'équation des ondes. Par suite

$$\Delta_0 \frac{\partial^k \rho_h}{\partial t_0^k} = \frac{\partial^{k+2} \rho_h}{\partial t_0^{k+2}} \tag{22}$$

Considérons l'équation (22) comme une équation de Poisson. Dans le cas où le second membre de cette équation est continu, les solutions de cette équation qui sont continues jusqu'au contour auront des dérivées premières continues. Donc, toutes les fonctions $\partial^k \rho_h / \partial t_0^k$ ont des dérivées continues du premier ordre. D'une façon analogue on peut déduire la continuité des secondes dérivées et ainsi de suite. Si la surface S est infiniment dérivable, toutes les fonctions ρ_h seront indéfiniment dérivables.

Notre théorème est démontré.

Corollaire 1. La valeur moyenne avec le poids ω d'une solution généralisée arbitraire est une fonction presque périodique de t.

Théorème 2. *Chaque solution de l'équation des ondes vérifiant la condition* (2) *et qui engendre une trajectoire de l'ensemble* M_1 *est une fonction presque périodique dans* L_2.

En effet, l'existence de l'intégrale $E_1(u)$ entraîne la continuité égale de la fonction $u(t)$ dans L_2. Cela étant établi, on peut déduire la presque périodicité de la fonction u de la presque périodicité de la fonction moyenne.

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ASTRONOMY

ON INCLINATION OF ORBIT PLANES OF VISUAL BINARIES TO GALACTIC PLANE

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The orientation of the orbit planes of binary stars with respect to the central plane of the Galaxy has attracted the interest of many authors. They have had good reason, indeed, to expect that the laws that might be discovered in such investigations would shed some light on the origin of these stars. But on account of the scarcity of the available data no definite conclusion has been reached; and, on the other hand, no one appears to have ever attempted a theoretical discussion of the problem.

In the present paper the author discusses the distribution of the inclinations of the orbit planes in the light of his theory on the origin of visual binaries. The observational data against which the distribution law derived by him may be checked, are published hereafter in a paper by S. V. Kozlovskaya ([2]).

1. According to the author's theory ([1]), binary stars—at least those which are visually observed to be such—have been formed by mutual capture from stars that formerly revolved independently of each other along their Galactic orbits.

In the above-cited paper the Galactic orbits of the two stars were assumed to be circles. This simplification had no other purpose but to make the geometric picture more illustrative without affecting in the least the validity of the conclusions derived. Now, however, more complicated phenomena are to be discussed, and therefore the problem must be treated in the general way, imposing no limitations upon the character of the Galactic orbits of the stars.

According to the hypothesis enunciated in the above-mentioned paper, capture occurs at the time when the stars are nearest to each other, being both simultaneously at the ends of the shortest distance between their respective Galactic orbits. The relative velocity L of the two stars at this instant is, on the other hand, the initial velocity in the relative orbit

$$L^2 = k^2 2M \left(\frac{1}{d} - \frac{1}{2a}\right)$$

where M is the total mass of the two stars, d is the shortest distance, and a is the semi-axis of the relative orbit. When the stars are in the position of shortest distance, the velocity L is perpendicular to the latter, therefore $d = a(1 \mp e)$, and we have

$$\frac{1}{1 \mp e} - \frac{1}{2} = \frac{}{\rho} \qquad (1)$$

where

$$\rho = \frac{k^2 2M}{L^2} \qquad (2)$$

It will be obvious that formula (1)—which coincides with formula (3) of the paper cited(1)—is valid irrespective of the qualifications made in that paper. The quantity ρ in the general case receives a more complicated expression.

If v and v_1 are the Galactic velocities of two stars, then

$$L^2 = v^2 + v_1^2 - 2vv_1 \cos\alpha$$

where α is the angle between v and v_1.

Put

$$\frac{L^2}{v^2} = 1 + \left(\frac{v_1}{v}\right)^2 - 2\frac{v_1}{v}\cos\alpha = \lambda^2$$

The rôle reserved to the quantity λ in our reasoning will later become apparent.

It is evident that in the particular case of circular orbits and with $v = v_1$ the new expression for ρ reduces to that referred to in the author's earlier paper(1), because then $\lambda^2 = 2(1 - \cos\alpha)$.

2. To proceed further in our investigation we need a definite system of coordinates. Let us choose it as follows. The X-axis will be directed along the velocity v_1 of one of the stars, the Y-axis will run perpendicular to it and parallel to the Galactic plane (its direction will be to the side where the centre of the Galaxy is located); finally, we define the Z-axis to be perpendicular to the XY-plane and directed to the northern hemisphere of the Galaxy.

As in their movement the stars, generally speaking, do not reach far out of the Galaxy, the X-axis all but lies in this plane (for the Sun the difference is 3° only). Accordingly, the Z-axis is very nearly directed towards the pole of the Galaxy. It is only in order to simplify our formulae as well as the operations to which we subject them, that we have fixed the system of coordinates to one of the stars instead of to the centre of the Galaxy.

Denote by α, β and γ the angles which the velocity v of the second star forms with the coordinate axis at the moment of capture. We shall then have the following angular coefficients for the two velocities

$$\begin{array}{llllll} v_1 & \ldots & 1 & 0 & 0 \\ v & \ldots & \cos\alpha & \cos\beta & \cos\gamma \end{array} \qquad (3)$$

As the direction of the shortest distance is perpendicular to either velocity, its angular coefficients are proportional to the minors of the above matrix: $0, -\cos\gamma, \cos\beta$.

The direction of the ralative velocity L is the geometric difference between v_1 and v. Therefore its angular coefficients are proportional to $v_1 - v\cos\alpha$, $-v\cos\beta$, $-v\cos\gamma$.

The direction of the pole of the relative orbit (let its angles with the axes be denoted by A, B and Γ) is perpendicular both to the shortest distance and to the relative velocity—two straight lines which, on our theory, determine the orbit plane. So its angular coefficients are proportional to the minors

$$v(\cos^2\gamma + \cos^2\beta);\ (v_1 - v\cos\alpha)\cos\beta;$$
$$(v_1 - v\cos\alpha)\cos\gamma$$

Ultimately the angular coefficients assume the following form

$$\cos A = \frac{\sin\alpha}{\lambda};\quad \cos B = \frac{\cos\beta\sqrt{\lambda^2 - \sin^2\alpha}}{\lambda\sin\alpha};\quad \cos\Gamma = \frac{\cos\gamma\sqrt{\lambda - \sin^2\alpha}}{\lambda\sin\alpha} \qquad (4)$$

where λ is the quantity afore-mentioned.

3. Our next problem is to find the distribution curve for the angle Γ—the inclination of the relative orbit to the plane of the Galactic motion of the star, which differs little from the plane of the Galaxy.

The probability of capture, according to our theory, is essentially characterized by the quantity ρ which varies inversely as λ^2. Therefore among the pairs formed those with very small λ (*i. e.* with small relative velocities) will predominate.

For the sake of concreteness adopt that 1) the quantity $\lambda = \frac{L}{v}$ is always small; so small, indeed, as to coincide practically with $\sin \lambda$; 2) the values of angle α are equiprobable within the narrow range of 0 to $\sin \alpha = \lambda$, to which they are confined according to formula (4); 3) taking account of Galactic rotation and of the special rôle played by the Galactic plane, we shall also assume that the values which angle β may possibly take and which all lie within $90° - \alpha$ to $90° + \alpha$ are equiprobable, too. Putting $\beta = 90° \pm \delta$, we have $\cos^2 \gamma = \sin^2 \alpha - \sin^2 \delta$ from the relation $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$. Substituting, in virtue of condition 1, the angles for their respestive sines and setting $\alpha = \lambda\xi$, $\delta = \lambda\eta$ we get

$$\cos^2 \Gamma = \frac{(\xi^2 - \eta^2)(1 - \xi^2)}{\xi^2} \quad . \tag{5}$$

The variables ξ and η have equal chances to take any value within their respective limits of definition, which are $0 \leqslant \xi \leqslant 1$, and $-\xi < \eta \leqslant \xi$. If ξ, η are taken for Cartesian rectangular coordinates, then with Γ constant the equation (5) represents a curve of order four, symmetrical with respect to the ξ-axis and having its real branch closed. The difference between the areas bounded by the curves with the parameters $\Gamma = \Gamma_1$ and $\Gamma = \Gamma_2$ characterizes the probability that choosing arbitrary values for ξ and η (within the above-mentioned limits) we shall hit the region between these curves. For a constant Γ the area of the figure bounded by the branch of the curve can be computed directly. The area above the ξ-axis (*i. e.* half of the total area) is

$$\frac{S}{2} = \int_{\xi=0}^{\xi=\sin\Gamma} \eta d\xi \quad .$$

where the limits correspond to the points at which η vanishes. The indefinite integral is known

$$\int \eta d\xi = \int \xi \sqrt{1 - \frac{\cos^2 \Gamma}{1 - \xi^2}}\, d\xi = \frac{\xi^2 - 1}{2} \sqrt{1 - \frac{\cos^2 \Gamma}{1 - \xi^2}} - \frac{\cos^2 \Gamma}{4} \ln \frac{1 - \sqrt{1 - \frac{\cos^2 \Gamma}{1 - \xi^2}}}{1 + \sqrt{1 - \frac{\cos^2 \Gamma}{1 - \xi^2}}}$$

Substituting the limits we have

$$S = \sin \Gamma + \cos^2 \Gamma \ln \operatorname{tg} \frac{90° - \Gamma}{2}$$

Note that for $\Gamma = 0$ we have $S = 0$, and for $\Gamma = 90°$ we have $S = 1$ so that the area is already normed.

The distribution function for the angle Γ is the derivative of S with respect to Γ

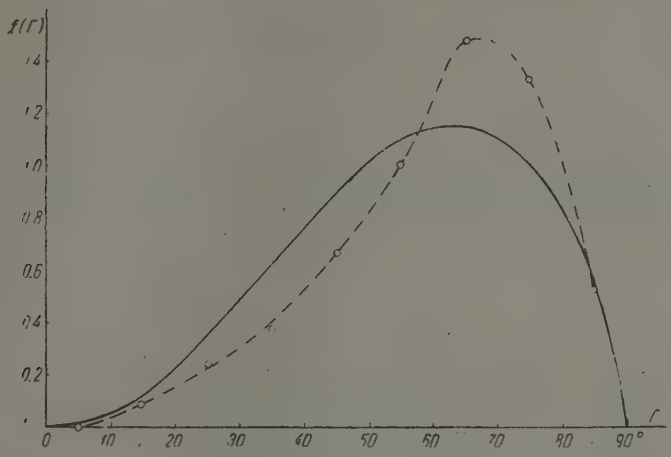
$$f(\Gamma) = \frac{dS}{d\Gamma} = \sin 2\Gamma \ln \operatorname{ctg} \frac{90° - \Gamma}{2} \tag{6}$$

On the figure this curve of distribution is represented by the solid line.

4. Comparison between the theoretical curve of distribution and the data based on observation is made difficult by the well-known fact that the angle Γ

cannot be obtained directly from observations, while its computation from the observed elements of the orbit gives two solutions i'_1 and i'_2, and which of the two is the true one remains undecided.

This difficulty, however, was ingeniously circumvented by S. V. Kozlovskaya (2), who computed the values i'_1 and i'_2 for all visual binary systems with known orbits, and then sorted out the stars for which the values i'_1 and i'_2 differ but little from each other. The broken curve in the drawing shows the factual distribution of i'



for these stars, according to Kozlovskaya. It appears from it that the distribution based on observations bears out all the theoretically predicted features of the distribution curve. A very slow rise up from zero at $i'=0$ is follwed by an abrupt decline down to zero near $i'=90$; the maximum of the observation curve lies close to the value predicted by the theory ($i'=64°$). Still it is by far more sharply expressed in the observation curve than in the theoretical curve. Whether the divergence is largely due to the simplification of the phenomenon by the theory, or whether it may be attributed to selection in the data, cannot be decided at this juncture.

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ASTRONOMY

ON THE DISTRIBUTION OF THE INCLINATIONS OF THE ORBIT PLANES OF VISUAL BINARY STARS TO THE PLANE OF THE GALAXY

By S. V. KOZLOVSKAYA

(Communicated by O. J. Schmidt, Member of the Academy, 6. VIII. 1945)

In order to decide in favour of one or another of the conflicting hypotheses that has been put forward to account for the origin of visual binary stars it is important to know, whether the angles at which their orbit planes are inclined to some fundamental plane of the universe display a regular or a random distribution.

However, in the way toward a successful solution of this problem stands, as has been pointed out by Aitken(1), the serious difficulty that in the general case the orbit of a binary star, computed only from visual observations, can be represented by two positions of a normal to it, *i. e.* by two poles, and for discrimination between the true and the spurious pole the elements of the orbit give no clue. Only when the inclination i of the orbit to a plane perpendicular to the line of sight is 0 or 90°, or when the sign of i has been ascertained by spectrographic observations, is the plane of the orbit, or its pole, defined uniquely. For the present there are only 22 visual binary stars for which the true orbit poles have been determined.

On the basis of different data—which are far from being abundant, indeed—different authors (1–11) have investigated the orientation of the orbit planes of visual binary stars to some particular plane of the universe. The results they have obtained for the distribution of the orbit poles can be boiled down to the following five groups.

1) Random distribution—Aitken (1), See (2), Doberck (4), Chang (9) Finsen (10).

2) Concentration along the Milky Way—Lewis and Turner (5).

3) Concentration near the pole of the Galaxy—Bohlin(6), Shajn(7), Hopmann(11).

4) Concentration near the pole of the ecliptic and the solar apex—Bohlin (6).

5) Concentration near the vertex of the preferential motion of the stars—Poor(8).

So the conclusions are seen to differ widely and even to contradict one another. This very circumstance was interpreted by some of the authors in favour of the random distribution. But it is safer to admit that the results obtained heretofore have left the question open.

A new approach to the problem has been found by O. J. Schmidt (12). Proceeding on a capture theory he derived by strict reasoning a distribution law which the inclinations of the orbit planes of the visual binary stars should obey in order to conform to his theory.

Now to check his results against observations more data were necessary than were actually available, and it was the object of the present investigation to secure

the additional material. I set out to compute the two possible inclinations of the orbit plane of a visual binary to the plane of the Galaxy, Γ_1 and Γ_2, for 211 binary pairs, using the data of the card catalogue of orbits, compiled by P. P. Parenago in the Sternberg Astronomical Institute. Seeing that even in the case of uncertain orbits the inclination i can be determined with sufficient confidence, I thought it feasible to increase the statistical material by taking in the bad orbits, too.

Let us consider spherical triangles connecting the pole of the heavens with the two possible poles of an orbit of a visual binary, and with the point of intersection between the line of sight to the star and the celestial sphere. The latter point is defined by the right ascension and declination of the star α, δ. Let α_1, δ_1 and α_2, δ_2, be the equatorial coordinates of the two possible poles of the orbit for the inclinations $+i$ and $-i$, respectively, and take $90-\Omega$ and $90+\Omega$ for the angles, formed by the circle of declination with the large circle passing through the star and through two possible poles of the orbit. Ω is the position angle of the node of the orbit. Without discriminating between the ascending and the descending node, we take the one whose position angle is less than 180° and describe it simply as «node».

The arc connecting the pole of the orbit with the intersection point between the line of sight to the star and the celestial sphere, is the inclination i.

From the spherical triangle we have

$$\sin\delta_1 = \sin\delta\cos i + \cos\delta\sin i\sin\Omega \qquad (1)$$

$$\sin\delta_2 = \sin\delta\cos i - \cos\delta\sin i\sin\Omega \qquad (2)$$

$$\sin(\alpha_1-\alpha) = \frac{\sin i\cos\Omega}{\cos\delta_1}\ ;\ \cos(\alpha_1-\alpha) = \frac{\cos i\cos\delta-\sin i\sin\delta\sin\Omega}{\cos\delta_1} \qquad (3)$$

$$\sin(\alpha_2-\alpha) = \frac{-\sin i\cos\Omega}{\cos\delta_2}\ ;\ \cos(\alpha_2-\alpha) = \frac{\cos i\cos\delta+\sin i\sin\delta\sin\Omega}{\cos\delta_2} \qquad (4)$$

Formulae (1), (3) and (2), (4) yield two pairs of values (α_1, δ_1) and (α_2, δ_1) for the equatorial coordinates of the orbit pole.

For these coordinates the Galactic latitudes b_1 and b_2 of the poles are found for every star in the Lund Observatory Tables ([13]). The Galactic longitudes of the poles are of no interest to us in the present case.

The required angles Γ_1 and Γ_2 are found as complements of b_1 and b_2.

In the 211 pairs of values thus obtained for the angles Γ_1 and Γ_2 the true inclinations to Galactic plane are intermixed with the spurious ones, and discrimination between them is only possible in the case of 22 stars.

However, for many stars the values of Γ_1 and Γ_2 closely approach each other. We can make use of this circumstance if we take into consideration that these values, in general, cannot be considered to be more accurate than up to 1° because of the error involved in the determination of the orbit elements i and Ω, upon which the values of Γ depend.

So, by sorting out, for instance, the stars for which the difference between the values of Γ_1 and Γ_2 is less than 10°, we can obtain a mean value $\Gamma_{mean} = \frac{1}{2}(\Gamma_1+\Gamma_2)$ differing from the true value of Γ (which is, of course, either the value of Γ_1 or of Γ_2) by less than 5°, and this inaccuracy will often lie within the error arising from the inaccurate determination of the orbit elements. Out of the 211 computed pairs 71 were fit to be thus selected. For these 71 stars a distribution curve was plotted giving the number n of stars against the inclination Γ to the plane of the Galaxy.

Denote by $|\Delta\Gamma|$ the deviation of Γ_{mean} from the true value of Γ. By allowing more room for $\Delta\Gamma|$ we can increase the statistical material and estimate the stability and characteristicity of the law which appeared when the error $|\Delta\Gamma|$ was more strictly limited.

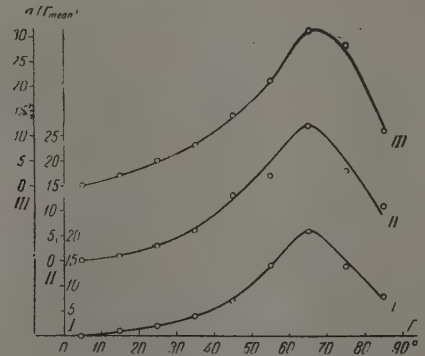
Thus, with $|\Delta\Gamma| \leqslant 7°.5$ the number of stars among the 211 computed increased to 96, and with $|\Delta\Gamma| \leqslant 10°$, to 120. For these groups of 96 and 120 stars the n (Γ_{mean})-curve was also plotted. In each of the three plots there were, of course, involved the 22 stars for which the true pole was known.

The distribution curves for $|\Delta\Gamma| \leqslant 5°$, $|\Delta\Gamma \leqslant 7°.5$, and $|\Delta\Gamma| \leqslant 10°$ are presented in the adjoined figure, while in the table are given the numbers of stars n (Γ_{mean}) separately for intervals of Γ_{mean} ranging from 0 to 10°, from 11 to 20°, and so on.

The Values of n (Γ_{mean}) for different ($\Delta\Gamma$)

Distribution curve \ Γ	0—10°	11—20°	21—30°	31—40°	41—50°	51—60°	61—70°	71—80°	81—90°	Total number of stars
I. For $(\Delta\Gamma) \leqslant 5°$	0	1	2	4	7	14	21	14	8	71
II. For $(\Delta\Gamma) \leqslant 7°.5$	0	1	3	6	13	17	27	18	11	96
III. For $(\Delta\Gamma) \leqslant 10°$	0	2	5	8	14	21	31	28	11	120

From the table, as well as from the figure, the distribution of the inclinations of orbits to the plane of the Milky Way appears to exhibit an unmistakable regularity, and its typical features can be traced in all three curves.



I, curve for 71 stars for which $|\Delta\Gamma| \leqslant 5°$; *II*, curve for 96 stars for which $|\Delta\Gamma| \leqslant 7°$; *III*, curve for 120 stars for which $|\Delta\Gamma| \leqslant 10°$

The curves begin from zero at $\Gamma = 0°$, rise slowly up to a maximum, clrearly expressed for Γ ranging from 60 to 70°, and then fall abruptly down to zero for $\Gamma \rightarrow 90°$.

The character of the curves agrees well with the distribution predicted theoretically by O. J. Schmidt ([12]). True, in the zero region our distribution curves run somewhat below the theoretical curve and rise higher than the latter in the region of maximum. But complete coincidence could hardly be expected considering the simplifications made by Schmidt in his discussion, and also on account of the small number of orbits used by me, which circumstance is further enhanced by possible distortions involved in the selection of stars with definite $|\Delta\Gamma|$, and finally, because the observed orbits may have failed to furnish a sufficiently representative material.

It should be noted that random distribution which holds advantage in the favour of several authors, is irreconcilable with the actual distribution as found

by me. In the case of random orientation the distribution must be proportional to sin Γ.

So the curve should rise smoothly as Γ grows from 0 to 90°, in sharp contrast with the curve derived from observations.

In order that the characteristic features of the actual distribution, as revealed in our curves, might be brought better to light, it is necessary to increase the quantity and quality of the observational data and to eliminate the influence of selection as far as possible.

The author desires to express his profound gratitude to O. J. Schmidt and P. P. Parenago for the helpful advice received in the course of this work.

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PHYSICS

ON THE FORMULATION OF THE HEISENBERG-BOHR INDETERMINACY PRINCIPLE FOR THE QUANTUM PROBLEM OF INTERACTION BETWEEN ELEMENTARY PARTICLES

By K. V. NIKOLSKY

(Communicated by S. I. Vavilov, Member of the Academy, 18. VII. 1944)

1. The number of elementary particles N_r is a dynamic variable in the same sense as an impulse or a unit vector of an individual particle is said to be one, and can be represented in the quantum formalism by means of a matrix N_r, expressible in different variables. If the state of the system is such that the number of particles is fixed by the initial conditions and is constant, then in describing this state in terms of the quantum formalism the matrix N is supposed to be diagonal, *i. e.* to have the form

$$(\alpha' | N_\alpha | \alpha'') = N_{\alpha'} \delta (\alpha' - \alpha'')$$

so that the number of particles enters into the number of dynamic variables by whose values the state of the dynamic system under consideration is characterized.

On the classical theory the number of particles is always considered to be quite definite, and by applying to it the quantum concept of dynamic variable we essentially widen the circle of phenomena which can be investigated on the principles of mechanics.

This, in fact, implies the existence of systems in such states in which the number of particles is an indeterminate quantity in the sense of the quantum theory. At the same time it should be noted that the usual method of quantum mechanics involves the concept of configuration space with a fixed number of dimensions, and is therefore unfit to deal with the above states. To make up this deficiency, the methods of quantum mechanics have been generalized to a new one described as «secondary quantization».

2. A dynamic system can be composed of particles of different type. It may consist, for instance, of heavy particles—protons and neutrons, and of light ones through which the interaction between electrically charged (electrons and positrons) and electrically neutral particles (mesons, photons, neutrinos) is materialized.

Accordingly, the number of characteristics of the system in the general form includes the numbers of particles $N_\alpha^{(1)}, N_\alpha^{(2)}, \cdots, N_\alpha^{(k)}, \ldots,$ which are dynamic variables not necessarily commuting with each other. Thus, a quantum dynamic system composed of particles of different type can be in a state where the number of particles of one type is fixed, while the number of particles of the other type, not commutable with the first, is indeterminate in the above-mentioned sense.

In this aspect we can consider the subdivision of a dynamic system into p a r t i c l e s and a f i e l d, by means of which interaction is transmitted from one

particle to another. It is the limiting case of this subdivision, corresponding to particles of great density, that is investigated in the classical theories of the field.

When additional degrees of freedom are introduced in investigating the distribution of energy in the system, the corresponding field is characterized by the probabilities of different distribution over their possible states of particles virtually describing the field.

So we can regard the subdivision of a dynamic system into field and particles as a subdivision into dynamic components corresponding to non-commutable assemblies of particles.

3. It is known that a relativistic problem cannot be consistently formulated unless it is made a problem of more than one body. And it may even be noted that in order to be consistent a relativistic quantum problem needs not merely to be formulated as a problem of n bodies, but also as one of the behaviour of a dynamic system formed by particles of different kind (in the sense of quantum mechanics).

The usual way to formulate the problem is at present as follows. The equations of motion are set forth for one particle in the external field, and the field is described, moreover, semi-macroscopically or macroscopically.

Such are Dirac's equations for an electron or a positron, the equations of Moller-Proc-Yukawa for a meson and Kemmer's system of equations, *etc.* In all these cases we have a problem of one body which is not in principle a relativistic way to put a problem. Artificially it is formulated as a problem of n bodies of relativistic nature, but this gives rise to difficulties connected with infinities of different kind.

Objections may also be raised against interaction between elementary particles being treated as the action of an external field on the particle, the field being described on the principle of correspondence. In order to be consistent from the relativistic point of view, in considering the problem we must consider an assembly of quantum particles of different type which has its properties determined by quantum principles, just as we do in the case of thermodynamic systems.

It will be of interest to follow up from this point of view the formulation of certain difficulties arising in the quantum theory of the field in connexion with the infinite values of the dynamic quantities characterizing an individual particle. It is well known that if in describing it in terms of quantum mechanics a relativistic particle is localized by means of a wave packet less than $\lambda_0 = h : mc^2$ in size, then the packet will contain components with energies of order mc. Thus, a wave packet describes an assembly of virtual particles rather than an individual particle.

In other words, it characterizes the state of a system with such a number of particles as is indeterminate in the sense of the quantum theory, because indeterminate in this sense is the number of transitions from one state to another.

From this standpoint Wick's arguments are of interest(1), which elucidate the quantum character of the interaction between elementary particles, and make it possible, in particular, to determine the range of action of the mechanical forces developing between the particles.

This is the way in which his arguments may be interpreted: interaction through particles implies that for the particles by means of which the interaction is materialized the notion of an individual particle, as determined by Heisenberg's inequalities, should be interpreted in terms of the problem of n bodies, with the number of particles undetermined in the sense of quantum mechanics.

4. An ordinary quantum dynamic system with a definite number of particles is described by means of a system of variables that are in relations of involution to each other. Similarly, quantum systems formed by particles of different kind can be characterized uniquely by canonic variables. It is most expedient to choose for variables $N_r(^k)$ and $\theta_r(^k)$ which are the numbers of particles in different alternative dynamic states, and the phases canonically

conjugated with them. The dynamic variables N_r and θ_r satisfy the generalized relations of involution, which present special cases of the relations

$$[A_r, A_s] = \sum_{\mu} c_{rs}^{\mu} A_{\mu}$$

where c_{rs}^{μ} are constants determining the group of transformations* characterized by the operators (matrices) $A_1, A_2, \ldots, A_{\mu}, \ldots$

In the case of dynamic systems where N_r' can take any value, $N_r' = 0, 1, 2, 3, \ldots$, the variables N_r and θ_r are connected by the relations

$$N_r e^{\frac{1}{\hbar}\theta_r} - e^{\frac{1}{\hbar}\theta_r} N_r = e^{\frac{1}{\hbar}\theta_r}$$

with $0 \leqslant \theta_r' \leqslant \hbar$ (the case of the so-called Boze-Einstein statistics).

The quantum dynamic variables $e^{\frac{1}{\hbar}\theta_r}$ are related directly to the probabilities of the particles of the assembly being redistributed over the possible states.

The variables θ_r are a quantum generalization of cyclic variables, of which the Hamiltonian of the system is independent.

For positrons and electrons N_r' can take only two values: 0 and 1, $N_r' = 0, 1$, and so the dynamic variable N_r can be represented by the matrix

$$N_r = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}_r$$

The generalized rules of commutation must be specialized accordingly, *i. e.*, the canonic variables to which N_r belongs, too, are represented by two-row matrices which can be obtained from the matrices σ_x, σ_y and σ_z

$$\sigma_x = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = i\sigma_x\sigma_y$$

by means of the general equation

$$\beta = a_0 + a_1\sigma_x + a_2\sigma_y + a_3\sigma_z$$

where a_0, a_1, a_2, a_3 are constants.

In this case the phase θ_r, as was proved by Jordan [2], can have only two values, *viz.* $\pm 1/2$, and so is an experimentally measurable quantity. It is a matter of considerable interest to follow up the relation of this circumstance to the spin of elementary particles.

On the other hand, the general rules of commutation can be specialized for other types of matrices N_r and for phases θ_r canonically conjugated with them, so that we have a new method for the systematization of the possible type of particles** (possessing higher spin values).

5. The mathematical formalism which makes it possible to describe the properties of quantum systems formed by particles of different kind may be regarded as a natural generalization of the quantum method when variables N_r and θ_r are used.

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* The quantum dynamic system is described by symbols A_α which form a kind of algebra of logics—a non-commutative linear algebra. The symbols A_α describe the results of measurement acts to which the quantum system under consideration can be subjected.

** We can formulate this poblem as one of determination of all irreducible representations of a given non-commutative algebra. To every irreducible representation there corresponds a particular type of particles with statistics of definite character and with definite phase relations.

Let us point out some features peculiar to this method. In the first place remember the relations that take place for an assembly of independent quantum systems. In this case the Hamiltonian of the system is a bilinear form in independent variables—generalized impulses. Therefore Schrödinger's equation coincides with Jacobi's equation from which it is obtained by passing from classical variables N_r and θ_r to non-commuting matrices (operators). Accordingly, the quantum properties of an assembly can be formulated in terms of geometrical optics.

In fact, the wave function describing an assembly has the form

$$\psi = \text{const}\, e^{iS/\hbar}$$

where S is the classical function of action expressed in the corresponding variables.

Thus, we have here what seems on first consideration to be a paradoxical state of affairs: transition to non-commuting variables does not carry us outside the realm of relations established by the classical mechanics of the phase space, in spite of the fact that the system considered enjoys all properties expressed by the principle of complementarity.

When the above method to formulate the problem is employed, the discreteness of the values of the dynamic variables is expressed through the discreteness of the values of N_r, and the initial conditions for the function of action S, *i. e.* through the corresponding phase relations.

The function of action S can be expressed explicitly through the dynamic variables N_r [3].

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PHYSIQUE DU GLOBE

L'ACCÉLÉRATION DE LA PESANTEUR DANS LE POINT GRAVIMÉTRIQUE PRINCIPAL DE TRANSCAUCASIE A TBILISSI

Par N. PARIJSKIJ

(Présenté par S. L. Soboleff, de l'Académie, le 3. XII. 1944)

Le système des points gravimétriques fondamentaux principaux de l'URSS comprend ceux de Poulkovo, de Moscou, de Poltava et de Kazan. Les accélérations de la pesanteur dans ces points, acceptées pour tous les travaux gravimétriques dans l'URSS, forment un système uni et réduit au système gravimétrique international de Potsdam. Le nivellement de toutes les 17 liaisons gravimétriques directes indépendantes de ces points entre eux-mêmes et l'Institut Géodésique de Potsdam, a été rempli par nous encore en 1935.

Après l'analyse, la réduction et l'évaluation de la précision de toutes les liaisons, on a obtenu des résultats suivants (sans arrondir le dernier chiffre):

Poulkovo	$g = 981.900_5$ gl	$\pm$ 0.54 mgl
Kazan	981.558_7	$\pm$ 0.68
Moscou	981.559_0	$\pm$ 0.74
Poltava	981.006_4	$\pm$ 0.67

A ce reseau fondamental n'était pas alors joint le point gravimétrique de Tbilissi, quoiqu'il fut lié directement à Potsdam par Gecker encore en 1909, mais ne possédant pas à ce moment de liaisons suffisamment neuves.

De même, il y avait alors de soupçonnements sur les changements temporaires de la pesanteur au Caucase. Les liaisons gravimétriques nouvelles de Tbilissi avec des points principaux fondamentaux, effectuées par l'Institut Seismologique de l'Académie des Sciences de l'URSS (par Léouchine en 1933—1934, et surtout par J. D. Boulanger au moyen de deux appareils à pendule), ont considérablement affermi la liaison de Tbilissi avec le réseau gravimétrique fondamental.

Il était très désirable pour les travaux gravimétriques dans la Transcaucasie d'y avoir un point fondamental local. Pour cela nous avons entrepris un grand travail pour reviser toutes les liaisons connues de Tbilissi au réseau fondamental, afin d'en déduire les valeurs de l'accélération de la pesanteur à Tbilissi. Dans ce but nous avons aussi utilisé, comme exception, trois liaisons, effectuées entre Tbilissi et le réseau fondamental par la Chambre principale des poids et mesures de Léningrade, étant donné que cette dernière possède une liaison sûre avec le système fondamental; étudié par Guigitsky (²).

L'étude de Guigitsky a été de même revisée par nous, et il a été admis pour la valeur du poteau fondamental de l'Institut de Métrologie et des Standards (WIMS) dans le système international $g = 981.932$ gl. ± 2.1 mgl, c'est-à-dire, de 0.5 mgl de plus qu'il en suivait des données de Guigitsky.

Après avoir effectué une analyse détaillée de toutes les 9 liaisons gravimétriques directes de Tbilissi a Potsdam avec des points fondamentaux de l'URSS, de même qu'avec l'Institut de Métrologie à Léningrade (WIMS), nous avons déduit là, où il a été possible de le faire avec une certitude absolue, les corrections systématiques, et nous avons réduit toutes les observations aux points fondamentaux. Les valeurs de la pesanteur dans le système international, pour le point fondamental gravimétrique de Tbilissi, trouvées par de différentes liaisons, sont données dans le Tableau 1. Nous y donnons de même les erreurs quadratiques moyennes, séparement pour chaque liaison, déterminées par la méthode que nous avons déduite, et évaluées de la façon aussi identique qu' il était possible envers des données si distinctes.

Tableau 1

L'accélération de la pesanteur à Tbilissi par de liaisons différentes dans le système international

Auteur	Année	Point de départ	g gl	ε mgl	Remarques
Chetkine	1903	Pulkovo, le sous-sol	980.178$_0$	±3.7	
Guekker	1909	Potsdame, 32	.178$_2$	±2.4	
Korsoune	1909	WIMS, le sous-sol	.175$_0$	±3.9	980.177$_5$*
Pavlov	1909	WIMS, » »			980.172$_4$**
Elistratov	1931	Poltava, » »	.177$_5$	±1.8	
Léouchine	1931	Poulkovo, la salle	.174$_5$	±4.3	
Léouchine	1933—1934	WIMS. le sous-sol	.174$_6$	±5.4	
Boulanger	1936	Moscou, Inst. Astron.	.173$_8$	±2.1	L'appareil de Cambridge
Boulanger	1936	Moscou, » »	.179$_4$	±1.6	L'appareil de Bamberg

* D'après les observations de Korsoune (Léningrade—Tbilissi).
** D'après les observations de Pavlov (Tbilissi—Léningrade) au moyen du même appareil à pendule.

La valeurs moyenne de la pesanteur à Tbilissi nous déduisons avec les poids $p = 1/\varepsilon^2$. A notre regret, les erreurs obtenues par nous et comprenant toutes les sources accessibles pour l'évaluation des erreurs, ne sont pas également uniformes. Cela se rapporte principalement aux cas où la liaison était unilatérale, et où il n'était pas possible d'évaluer l'erreur systématique, commune au point fondamental, comme, par exemple, pour les liaisons de 1936. Pour cette raison nous donnons aussi, pour la comparaison, les moyennes avec les poids $p' = 1/\varepsilon$ et sans poids.

Les valeurs moyennes d'accélération de la pesanteur à Tbilissi sont les suivantes (Tableau 2).

Tableau 2

Les valeurs moyennes de l'accélération de la pesanteur à Tbilissi

	Sans poids		$p' = 1/\varepsilon$		$p = 1/\varepsilon^2$	
	g gl	ε mgl	g gl	ε mgl	g gl	ε mgl
Les déterminations anciennes de 1903—1909	980.177$_1$	±2.0	980.177$_3$	±1.8	980.177$_5$	±1.8
Les déterminations neuves de 1931—1936	.176$_0$	±1.2	.176$_6$	±1.1	.177$_1$	±1.0
La moyenne des déterminations anciennes et neuves	.176$_6$	±1.2	.177$_0$	±1.0	.177$_2$	±1.0
Toutes les déterminations . . .	.176$_4$	±1.2	.176$_8$	±0.9	.177$_2$	±0.8$_4$

Ainsi, pour un nouveau point fondamental à Tbilissi (l'avenue de Plekhanoff, 150, La station seismique de l'Académie de Géorgie, le corps principal, sous-sol de l'est, le grand poteau No. 8)

$$\varphi = 41°43'08'', \quad \lambda = 44°47'42'' E; \quad h = 400.7\, m$$

nous avons obtenu

$$g = 980.177_2\, \text{gl} \pm 0.8_4\, \text{mgl}$$

La différence entre les nouvelles et les anciennes déterminations, avec la différence des époques de 30 années, se trouve hors de limite de précisions des déterminations pendulaires de la pesanteur.

Les anciennes déterminations de 1903—1909 donnent

$$g = 980.177_5\, \text{gl} \pm 1.8\, \text{mgl}$$

Les nouvelles déterminations de 1931—1936 donnent

$$g = 980.177_1\, \text{gl} \pm 1.0\, \text{mgl}$$

$$g_{\text{neuve}} - g_{\text{anc}} = -0.4\, \text{mgl} \pm 2.1\, \text{mgl}$$

Nous pouvons, par conséquent, conclure que la pesanteur à Tbilissi, à la précision des mesures pendulaires près, n'a pas changé au cours des 30 années par rapport au système des points principaux fondamentaux.

Donc, le sous-sol de la station seismique de Tbilissi peut être employé comme un nouveau point gravimétrique de l'URSS. pour lequel l'accélération de la pesanteur est obtenue avec la précision presque la même que pour les autres quatre points fondamentaux de l'URSS.

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Académie des Sciences de l'URSS.

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CHEMISTRY

AN ELECTROLYTIC METHOD OF SEPARATING NICKEL AND COBALT

By K. A. NENADKEVICH

(Communicated by W. I. Vernadsky, Member of the Academy, 12. XI. 1944)

Among the methods proposed for the separation of nickel and cobalt a considerable part is based on the greater stability of cobalt complexes as compared to nickel complexes, and also on the fact that these elements have a different valence in analogous compounds, cobalt being trivalent and nickel bivalent. The so-called hypochlorite method in the separation of nickel and cobalt cyanide complexes and Liebig's method of separating the same cyanide complexes by means of mercuric oxide are based on this principle.

The present author has also employed it in working out a new analytical method whereby these two elements are separated by precipitating cobalt with silver nitrate in the form of the complex salt silver cobalticyanide, which is more stable to the action of acids and bases than the analogous complex salt of nickel. The desired result is achieved by a preliminary treatment of the cyanide complexes of cobalt and nickel with nitric acid. In this way the nickel complex is completely decomposed to a simple salt, and subsequent precipitation with silver nitrate gives the silver-cobalt complex of the indicated salt $Ag_3Co(CN)_6$.

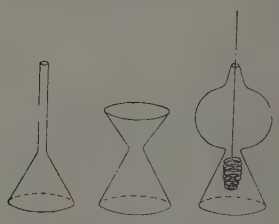
Until now, however, there has been no mention of an electrolytic method based on the greater stability of the cobalticyanide complex as compared with the analogous nickel salt. The present work is an outcome of numerous experiments carried out in this direction, and summarizes in brief the results obtained and the separation procedure itself.

In the course of preliminary experiments the cyanide complexes were observed to behave differently during electrolysis in alkali solutions (potassium hydroxide and sodium hydroxide). Whereas the nickel complex decomposes and black nickel oxide is deposited on the anode, only a change in the colour of the solution from lemon-yellow to orange is observed in the case of the cobalt complex, even after many hours of electrolysis.

In both cases very weak deposit of metallic nickel and cobalt appears on the cathode. Thus, if such conditions could be provided under which nickel and cobalt could be prevented from depositing on the cathode, it would be possible to separate nickel completely in the form of the black oxide on the anode, and leave all the cobalt in the electrolyte. To provide these conditions use was made of the depolarizing action of the potassium and sodium salts of chromic acid. A series of preliminary experiments showed that this method of separation is so effective that it is possible quickly and without difficulty to establish the presence of nickel in cobalt preparations even without employing the depolarizer mentioned above, if the purity of the preparation needs only to be appraised qualitatively.

A quantitative method, of course, requires the use of a depolarizer to prevent both metals from depositing on the cathode.

The depolarizing action of the chromic salts is due to the oxidation process on the cathode accompanied by the reduction of chromic acid to chromium oxide which with the alkali present gives a chromite that in turn oxidizes on the anode to the initial chromate. In the given case the depolarizer acts as a catalyst, whose rôle consists in preventing nickel and cobalt from depositing on the cathode until all the nickel has been deposited on the anode in the form of the black oxide. The electrolyte used was obtained by precipitation followed by dissolution of the total salt of nickel and cobalt (the sulphates are best of all) with the least possible excess of potassium cyanide. After this the solution is heated and evaporated on a boiling water bath for no less than an hour to guarantee the complete auto-oxidation of the cobalt salt, which is indicated by the solution becoming light yellow and, in the case of excess KCN, nearly colourless. To make sure that the oxide of the salt is obtained, some authors [1] advise evaporating to dryness and then dissolving the dry precipitate. The volume of the electrolyte in our experiments was about 50—60 ml; the capacity of the platinum dish about 100 ml; the total weight of the cobalt and nickel salts taken for electrolysis was about 0.5 g; 2 g of potassium hydroxide were added to the electrolyte.



Three types of electrolytic cells of the open type: *1*, a simple funnel; *2*, a double paired funnel, *3*, a funnel with a globe

The electrolytic process roughly proceeds as follows: several minutes after switching on the current (from 0.1 to 2 A at 2—3 V) a black ring of nickel oxides appears on the anode (platinum dish), and then evolution of oxygen and hydrogen begins on the poles of the bath. After a certain period of time, the length of which is determined by the excess of potassium cyanide present in the electrolyte (during the formation of the cyanide complex), the dish, upon the complete decomposition of the latter (to cyanate), begins to be covered evenly by black nickel oxide which, if present in considerable amounts, is not very firmly attached to the anode. In this case we nearly always observe the parallel formation of the light green nickel protoxide which is the first product of the alkali. As electrolysis proceeds, the green nickel protoxide is oxidized to the black oxide and remains on the bottom of the dish in the form of a dense easily filterable and washable precipitate. When electrolysis is over, the current is shut off, the electrolyte containing all the cobalt in the form of the initial cyanide complex plus the added depolarizer is separated by filtration, and the washed filter with the black oxide is placed into the electrolytic dish covered by the rest of the black oxide, and the dish is annealed at a red heat until the green nickel protoxide is formed which is then weighed together with the dish.

When it is desired to determine nickel by some other method, the filter with the precipitate that has been transferred to the dish and the nickel oxide remaining on the latter are treated with nitric acid, and, after filtering, the solution is used to precipitate the compound desired for weighing and determining nickel.

The cobalt in the filtrate from the nickel, which contains all of it plus the depolarizer (the added chromate), is precipitated (after the potassium hydroxide present has been neutralized with nitric acid and the solution has been slightly acidified) with silver nitrate in the form of the double silver and cobalt cyanide complex [2].

In carrying out the above experiments an electrolytic cell of the open type was used to separate the gases evolved at the cathode and anode. It consisted of a funnel passing over into a globe and suspended from the spiral cathode. Applicable, though less satisfactory, are two other types of open cell *1* and *2* (see figure). These are less convenient, however, because they do not provide against spattering of the electrolyte through the exit opening of the cell. At the narrow neck where

the funnel passes over into a globe the cell was covered by a watch glass of an ordinary piece of glass that had been cut in half and had a hole in the centre so that it could be fitted into place. The dish was always used as anode. According to our observations, the proposed cell facilitates rapid and accurate electrolytic separation.

The principles lying at the basis of the outlined method are at present being investigated in our Institute with a view to applying them to the electrolytic separation of the entire series of elements in the group containing iron, manganese, cobalt, nickel and chromium, when two of them are present simultaneously.

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CHEMISTRY

A GRAVIMETRIC ANALYTICAL METHOD OF SEPARATING COBALT FROM NICKEL IN CYANIDD COMPLEXES

By V. S. SALTYKOVA

(Communicated by W. I. Vernadsky, Member of the Academy, 12. XI. 1944)

Several methods are available for separating nickel and cobalt. Most important are those based on the nitrite, nitrosobetanaphthene, glyoxime, hypochlorite; and of the colorimetric methods, that involving rhodanide-acetone must be mentioned in the first place.

The purpose of the present research was to simplify and improve the existing methods without detriment to the accuracy of the results. The difference between cobalt and nickel in their response to certain reagents is most striking when the elements are in different valence states. This is the case, for example, in the formation of complex cyanides of cobalt and nickel, where cobalt is present in the trivalent state and nickel in the bivalent, which facilitates their separation.

When a mixture of neutral salts of cobalt and nickel is treated with a solution of potassium cyanide, simple cyanides of these metals are first formed which, upon further addition of an excess of the precipitating agent, dissolve completely. In this way complex soluble salts—potassium cobalticyanide and potassium nickel cyanide are formed:

$$Co(NO_3)_2 + 2KCN = Co(CN)_2 + 2KNO_3$$
$$Ni(NO_3)_2 + 2KCN = Ni(CN)_2 + 2KNO_3$$
$$Co(CN)_3 + 4KCN = K_4Co^{\cdot\cdot}(CO)_6$$
$$Ni(CO)_2 + 2KCN = K_2Ni^{\cdot\cdot}(CN)_4$$

When the solution is left standing in contact with air, the nickel complex does not undergo any further change. The cobalt complex, on the other hand, undergoes auto-oxidation by the oxygen of the air, and yields an oxide salt in which the cobalt is already trivalent. Similar oxidation may take place owing to the dissociation of water as well:

$$2K_4Co^{\cdot\cdot}(CN)_6 + H_2O + O = 2K_3Co^{\cdot\cdot\cdot}(CN)_6 + 2KOH$$

or

$$2K_4Co^{\cdot\cdot}(CN)_6 + 2H_2O = 2K_3Co^{\cdot\cdot\cdot}(CN)_6 + 2KOH + H_2$$

Herewith the solution changes in colour from green to straw-yellow.

After the cobalt has been oxidized, a distinct difference is observed in the chemical properties and stability of both complexes. True, neither of them are affected by alkalies, but it is quite a different matter when it comes to acids. The nickel complex is decomposed by any mineral acid even in the cold, giving the simple

nickel salt of the acid, whereas the cobalt complex decomposes only upon evaporation with concentrated sulphuric or chloric acids. Furthermore, the cobalt complex is not affected by nitric and hydrochloric acids. Salts of the heavy metals react by double decomposition with potassium cobalticyanide and potassium nickel cyanide, yielding salts that are stable with respect to acids in the case of cobalt and slightly stable in the case of nickel. Thus, upon the addition of silver nitrate or mercurous nitrate to an acid solution of potassium cobalticyanide, a bulky, yellow, floculent precipitate of silver cobalticyanide, $Ag_3Co(CN)_6$, or mercury cobalticyanide, $Hg_3Co(CN)_6$, respectively, settles, which is insoluble in water and weak acids.

When both the nickel and the cobalt cyanide complexes are present simultaneously in the solution, the former, upon treatment with acid, forms nickel cobalticyanide which is stable with respect to acids, but enters into double decomposition with salts of the heavy metals should they be present.

$$2K_3Co(CN)_6 + 6AgNO_3 = 2Ag_3Co(CN)_6 + 6KNO_3$$

$$2K_3Co(CN)_6 + 6HgNO_3 = 2Hg_3Co(CN)_6 + 6KNO_3$$

$$2K_3Co(CN)_6 + 3K_2Ni(CN)_4 + 12HNO_3 = Ni_3[Co(CN)_6]_2 + 12HCN + 12KNO$$

$$Ni_3[Co(CN)_6]_2 + 6AgNO_3 = 2Ag_3Co(CN)_6 + 3Ni(NO_3)_2$$

The complex salt of silver and cobalt that forms in neutral and weakly acid solutions has a constant composition ([1]), fits the formula, and from the standpoint of insolubility([2]) is a convenient form for the gravimetric determination of cobalt. The fact that nickel under the same conditions forms a nickel complex that is easily decomposed by acids makes it possible to determine cobalt in its presence. It is this reaction that has been made the basis of the method proposed below.

The reaction yielding the silver salt of cobalticyanic acid is referred to in the works of several authors. Rodgers ([1]), Zwenger([2]), and Miller and Mathews give the formula and method of formation of silver cobalticyanide and describe its properties. In 1931 Czaporowski and Wiercinski([4]) made the reaction the basis for a potentiometric method of determining silver.

The reaction was applied to separate nickel when cobalt is simultaneously present by Sanchez([5]) in 1909, but in the method he proposed cobalt is not determined directly, but is computed from the difference after determining the total content of both elements, and herein lies the essential difference between his method and the one proposed here. The former can be employed for the determination of nickel only, is insufficiently precise, and very cumbersome into the bargain.

In the method evolved by the author in collaboration with K. A. Nenadkevich, the separation and determination of nickel and cobalt is carried out as follows. To a neutral or weakly acid solution of a mixture of nitrate (not chloride!) salts of cobalt and nickel in a porcelain dish is added a solution of potassium cyanide until the precipitate that forms at first is completely dissolved. The solution in the dish is evaporated to dryness on a water bath after which the dry residue is treated with water. Evaporation to dryness is necessary in order to be sure that the cobalt has completely passed over into the trivalent state. To the resulting solution of cyanide salts are added 10 ml of HNO_3 (sp. gr. 1.42), and it is again evaporated in the same dish nearly to dryness. Herewith the excessive potassium cyanide is expelled and completely decomposed, and the nickel complex breaks up forming the simple nitrate salt of the metal. Liberated from the comlex anion, the nickel immediately reacts with the cobalticyanide complex forming a precipitate of the nickel salt of cobalticyanic acid of a characteristic greenish-blue colour, corresponding to the formula $Ni_3[Co(CN)_6]_2$. During the second evaporation the hydrocyanic acid formed during the disintegration of the nickel complex is also completely expelled. This is essential because otherwise silver cyanide precipitates with the further addition of silver nitrate, which distorts the experimental results. The nearly dry residue that is obtained is treated with hot water,

and then 20 ml of a 5 per cent silver nitrate solution are gradually added to the solution.

In this reaction a double decomposition takes place between the complex salt of nickel cobalticyanide and silver nitrate, and insoluble silver cobalticyanide is formed, the final form in which the cobalt is weighed, the colour of the precipitate changing from blue to cream. Then the dish fitted with a cover glass is heated for an hour on a water bath, the precipitate is allowed to settle, and the latter is filtered through a weighed Shott crucible. The precipitate is now thoroughly washed with hot water and without removing it from the crucible dried at 130° to constant weight, in the course of which it becomes slightly darker. Then the crucible is weighed and the weight of $Ag_3Co(CN)_6$ recomputed in terms of metallic cobalt. The inversion factor is 0.1146.

A big advantage of the proposed method of analysis is that it does away with intermediate filtration and transference of solution, the entire process being carried out in one and the same dish. The nickel that remains in the filtrate may be determined by any of the known methods, after the excessive silver has been removed in the form of the chloride by means of hydrochloric acid.

A version of the method was also tested in which, after the stage where nickel cobalticyanide is obtained, this precipitate, owing to the ease with which it reacts with alkalies, was treated with potassium or sodium hydroxide, this resulting in the formation of a hydrate of nickel oxide that may be directly filtered off, the cobalt passing over into solution in the form of $K_3Co(CN)_6$ and being capable of precipitation by silver after the solution has been acidified. However, a more precise investigation of the hydrate of nickel oxide obtained showed that it always contains small amounts of cobalt, and that is why this version was not adopted in the final development of the method.

Perhaps, after suitable changes in the analytical procedure, this version will prove useful in cases where nickel has to be determined before cobalt, or just alone when both are present. Work in this direction is being continued.

Mercury salts give a precipitate similar to silver salts. The use of mercury salts would have noticeably simplified the analysis, making it possible to determine cobalt directly in the form of Co_3O_6 by simple calcining, on the one hand, and on the other, by similar calcining of the precipitated total sulphides of nickel and mercury in the filtrate, to obtain pure nickel oxide which is directly weighed.

In this connexion experiments were set up to elucidate the possibilities of replacing the silver salts by mercury salts. These experiments showed, however, that mercury cobalticyanide, under the conditions necessary for the separation, is considerably more soluble than the corresponding silver salt, and for cobalt underestimated results are always obtained.

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PHYSICAL CHEMISTRY

DISPLACEMENT OF THE IGNITION LIMITS FOR CARBON MONOXIDE BY SMALL ADMIXTURES OF HYDROGEN

By V. N. KONDRATIEV, Corresponding Member of the Academy

It has been shown by M. Ashkinazi in our laboratory that small admixtures of hydrogen (a few hundredths of a per cent) extend the region of combustion of oxygen mixtures of carbon monoxide, lowering the lower limit of ignition (p_1) and raising the upper one (p_2). It is easy to show that this effect of hydrogen ensues from the mechanism of the combustion of CO. On the basis of this mechanism [1] the condition of the ignition limits (defined by the equality of the rates of the branching and rupturing chains of the processes) may be written in the form

$$k_2(O_2)(H) + k_5(H_2)(O) = k_3(M)(O_2)(H) + k_6(H) + [k_{18}(CO) + k_{19}(O_2)](M)(O) \quad (1)$$

where k_2, k_3 . . . are the rate constants of the respective processes, and (O_2), (CO) . . . , the concentrations. Employing the method of partial stationary concentrations and assuming that rupture of the chains on the walls in the case under consideration ($p_1 > 10$ mm) is limited by diffusion of the active centres [$k_6 = k_6^0/(M)$], we may reduce equality (1) to the form

$$(Ap + \varepsilon)(Cp^3 + B) - \varepsilon p^2 = 0 \quad (2)$$

Here p stands for the limit pressure of the mixture, and ε, the relative concentration of hydrogen

$$\varepsilon = \frac{p_{H_2}}{p}$$

while A, B and C are constants which, in the case of an equimolecular mixture, are expressed by the formulae

$$A = \frac{k_{18} + k_{19}}{2k_5}, \quad B = \frac{k_6^0}{k_2}; \quad C = \frac{k_3}{2k_2}$$

At the lower ignition limit, Cp^3, being small in comparison with B, may be neglected, and equality (2) rewritten in the form

$$(Ap + \varepsilon)B - \varepsilon p^2 = 0$$

whence it follows that

$$p_1 = \frac{AB}{2\varepsilon}\left(1 + \sqrt{1 + \frac{4}{A^2B}\varepsilon^2}\right) \quad (3)$$

i. e. that there is a lowering of the lower limit with an increase in ε.

It is just the opposite at the upper limit. Here B is small in comparison with Cp^3 and may be neglected, which gives

$$(Ap+\varepsilon)Cp-\varepsilon=0$$

whence we get

$$p_2=\frac{\varepsilon}{2A}\left(\sqrt{1+\frac{4A}{C\varepsilon}}-1\right) \qquad (4)$$

i. e. a rise in the upper limit with an increase in ε.

We will show that expressions (3) and (4) not only correctly describe the observed facts qualitatively, but give values for the constants A, B and C that are of the correct order of magnitude.

Compare Ashkinazi's experimental data for p_1 at different ε (equimolecular mixture, $t=650°$ C) given in the first column of table 1 with the calculated results in the second column obtained with formula (3), when the constants A and B have the following values: $A=0.9\cdot10^{-5}$ mm^{-1}, $B=100$ mm^2.

Table 1

p_1 (obs.) in mm	p_1 (calc.) in mm	ε %
20.8	20.7	0.00625
20.0	19.8	0.00665
17.8	15.6	0.010
12.0	12.4	0.021

Table 2

p_2 (obs.) in mm	p_2 (calc.) in mm	ε %
32.5	31.6	0.012
35.5	36.5	0.020
38.0	38.5	0.025
42.5	41.0	0.032

The quantities A, B and C may also be computed from the following values of the constants entering into them

$$k_2=6.2\cdot10^7\frac{10^{-3920/T}}{\sqrt{T}}\ \text{mm}^{-1}\,\text{sec.}^{-1}\,(^2),\qquad k_3=\frac{1.65\cdot10^5}{T^2}\ \text{mm}^{-2}\,\text{sec.}^{-1}\,(^2)$$

$$k_3\cong3.10^8\frac{10^{-1320/T}}{\sqrt{T}}\ \text{mm}^{-1}\,\text{sec.}^{-1}\,(^1),\quad k_6^0=\frac{1}{3}T^3/_2\,\text{mm}\,\text{sec.}^{-1}\,*$$

$$k_{19}\cong40k_{18}\cong1.4\cdot10^5\frac{10^{-875/T}}{T}\ \text{mm}^{-2}\,\text{sec.}^{-1}\,(^1)$$

From these data we find for $T=923°$ absolute that $A\cong2.4\cdot10^{-5}$ mm^{-1} (instead of $0.9\cdot10^{-5}$ mm^{-1}), $B=90$ mm^2 (instead of 100 mm^2), and $C=0.94\cdot10^{-3}$ mm^{-1}.

It should be noted that owing to the insufficient accuracy of the constants k_5, k_{18} and k_{19}, here it can be a question only of the order of magnitude of the constant A. That is why the agreement between the values of A found in the two different ways (from formula (3) and by calculation) should be regarded as fully satisfactory. Computing further the quantity $Cp_1^3\leqslant\leqslant0.94\cdot10^{-3}\cdot21^3\cong8$, and comparing it with B, we may become convinced of the correctness of the assumption we made above that $Cp_1^3\ll B$.

In Table 2 we cite experimental data for p_2 (equimolecular mixture, $t=$ $=540°$C) and, for comparison, calculated results obtained with formula (4)

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* The constant k_6^0 is computed from $\frac{1}{k_6}=\frac{3\pi}{4}\frac{x^2}{v^2}Z$ for $x=1$ cm, and

$$Z=2\sqrt{\frac{2}{\pi}}\,\sigma\,\sqrt{\frac{RT}{\mu}}\,\frac{p}{kT}\ \text{when}\ \sigma=3\cdot10^{-15}\ \text{cm}^2.$$

when the constants A and C have the following values: $A = 0.32 \cdot 10^{-5}\,\mathrm{mm}^{-1}$, $C = 17.3 \cdot 10^{-3}\,\mathrm{mm}^{-1}$. Computing these quantities from the k constants given above, we find that $A \cong 2.9 \cdot 10^{-5}\,\mathrm{mm}^{-1}$ (instead of $0.32 \cdot 10^{-5}\,\mathrm{mm}^{-1}$), and $C = 38 \cdot 10^{-3}\,\mathrm{mm}^{-1}$ (instead of $17.3 \cdot 10^{-3}\,\mathrm{mm}^{-1}$). The poorer agreement here between the calculated and observed values of A and C (especially A) in comparison with the previous case must evidently be ascribed to the fact that the approximate expression (4) is less accurate than (3). This is actually born out by calculation of B and Cp_2^3 for $t = 540^\circ$ C, which gives $260\,\mathrm{mm}^2$ and $595 - 1330\,\mathrm{mm}^2$, respectively, whence it follows that the inequality $Cp_2^2 \ll B$ is observed only very approximately here.

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TECHNOLOGIE CHIMIQUE

SUR LA DÉSILICATISATION AU COURS DE L'ÉPURATION DE L'EAU D'ALIMENTATION DE SCHAUDIÈRES A VAPEUR

Par J. J. LOURIER et V. A. KLIACHKO

(Présenté par I. I. Černjaiev, de l'Académie, le 30.XI.1944)

L'entartrage d'ébullition des parois de chauffe des chaudières à vapeur est habituellement constitué par le dépôt de carbonates de calcium et de magnésium, de sulfates de calcium et de silicates de calcium et de magnésium. Ces derniers forment les dépôts les moins compacts, particulièrement difficiles à éloigner et d'une conductibilité thermique particulièrement basse. Ces particularités des dépôts de silicates expliquent le fait que la présence d'acide silicique dans l'eau d'alimentation des chaudières à vapeur provoque une diminution de la sûreté de travail de ces dernières par suite de l'apparition possible de boursouflures, de fissures et d'éclatement des tuyaux, et entraîne une augmentation de la consommation de charbon et des frais que nécessite l'éloignement du tartre.

La meilleure méthode d'éviter les dépôts de silicates sur les surfaces de chauffe des chaudières à vapeur consiste à débarasser le plus complètement possible l'eau d'alimentation de l'acide silicique. Cette méthode est l'unique possible pour éviter l'entartrage d'ébullition dans les chaudières monocuvélaires dans lesquelles toute l'eau d'alimentation se transforme en vapeur, ce qui fait que l'on ne peut employer des méthodes comme la phosphatisation, l'alcalinisation et autres, qui sont largement utilisées dans les chaudières cylindriques et qui consistent à transformer les sels, susceptibles de donner des dépôts, en scories, qui sont alors éloignées par l'eau de nettoyage.

On a recours actuellement dans la pratique d'épuration de l'eau d'alimentation des chaudières à vapeur aux méthodes de désilicatisation suivantes:

a) une méthode chimique consistant à transformer le SiO_2 contenu dans l'eau en silicates peu solubles de Ca, Mg ou de Fe;

b) une méthode physico-chimique se basant sur un passage à l'état colloïdal de l'acide silicique en dispersion moléculaire et sur la coagulation consécutive du colloïde obtenu par les sels d'aluminium ou de fer.

Ces deux méthodes n'ont pas trouvé jusqu'ici de large application pratique, car la première ne donne pas de résultats satisfaisants par suite de la solubilité considérable de $CaSiO_2$, même en présence d'un excès de substance précipitante; et la seconde méthode demande un appareillage encombrant, une grande dépense de réactifs. Un autre défaut de cette méthode consiste dans la difficulté d'obtenir une désilicatisation poussée dans les conditions d'exploitation industrielle; il se montre en effet comme impossile de maintenir fixe le point isoélectrique de coagulation par suite des variations continuelles dans la qualité de l'eau d'alimentation.

Brianski[1] a montré que pour une zone étroite des valeurs isoélectriques (pour un pH compris entre 7.2 et 7.6) il se montre possible d'abaisser la teneur de l'eau

en acide silicique de 40 mg/l jusqu'à un taux de l'ordre de 0.5—2.0 mg/l, en traitant l'eau par l'aluminate de sodium tout en corrigeant son pH par l'acide chlorhydrique et d'hydrate de chaux. La méthode nécessite une très forte dépense d'aluminate de sodium—200 mg/l, et le résultat obtenu laisse nettement à désirer quand le pH de l'eau tombe au-dessous de 7.2 ou dépasse 7.8.

Schwarz[2] put diminuer la teneur en SiO_2 en traitant par $Fe_2(SO_4)_3$ l'eau qu'il maintenait entre les mêmes limites étroites de la zone isoélectrique. Dans les deux cas l'effet de désilicatisation était atteint de 1.5 à 2 heures après l'emploi des réactifs.

Les auteurs de cet article ont mis au point une méthode permettant la désilicatisation de l'eau d'alimentation au cours de son épuration chimique habituelle. Le principe en est le suivant: comme dans les installations habituelles d'épuration chimique, l'eau est envoyée sur un filtre, où elle traverse des substances qui fixent les cations qu'elle contient et les remplacent par des ions H^+. Sortant du filtre, cette eau «hydrogénisée» ne contient plus de cations Ca, Na ou Mg et présente une réaction acide. Dans cette eau acidifiée, pourrait-on dire, on ajoute en la dosant une quantité de HF ou de ses sels, un peu supérieure à celle que nécessiterait la réaction

$$H_4SiO_4 + 6HF = H_2SiF_6 + 4H_2O$$

Il se produit partiellement au cours de cette réaction du fluorure de silicium volatil. En faisant traverser l'eau par un courant d'air on favorise la décomposition de H_2SiF_6 à l'état de HF et de SiF_4 et l'élimination de SiF_4. L'acide fluorhydrosilicique, restant en solution, est un acide fort; sa constante de dissociation en solution normale et à 25° C est en effet de 0.53[3], et on obtient son élimination en filtrant l'eau à travers un filtre contenant des produits capables de fixer les anions Cl', SO_4'', SiF_6'' et de les remplacer par l'ion OH'. Les avantages de la méthode proposée consistent dans l'économie de son emploi, dans sa concordance avec la méthode habituelle d'épuration chimique de l'eau d'alimentation des chaudières, dans la possibilité d'utiliser avec cette méthode l'appareillage d'épuration habituel, sans lui faire subir de grands changements.

Il fallait cependant contrôler l'évolution dans le temps du processus chimique proposé dans le cas de très faibles concentrations de SiO_2, vérifier la dépense des réactifs et la possibilité d'employer pour l'élimination de SiF_6'' du goudron synthétique habituel, communément employé comme fournisseur d'ions OH' dans les installations d'épuration chimique.

On procèda à cet effet aux expériences suivantes. On ajoutait à de l'eau distillée 35—200 mg/l de SiO_2 sous forme de Na_2SiO_3 et 600 mg/l de HCl, ce qui correspond à l'acidité après saturation par cations H d'une eau de dureté totale de 50° H.

On ajoutait ensuite à l'eau l'acide fluorhydrique, ou ses sels de sodium ou de calcium, en quantité une fois et demi supérieure à la quantité que nécessiterait la réaction

$$H_4SiO_4 + 6F' + 4H^+ = SiF_6'' + 4H_2O$$

Après avoir ajouté F', on faisait passer dans l'eau un courant d'air pendant dix minutes. On filtrait ensuite la solution avec la vitesse de 2 m/h en se servant d'un filtre de laboratoire en verre, rempli d'une couche de 35 cm de goudron de metaphénylenediamine, preparé d'après la méthode d'A. S. Smirnov. Le goudron était obtenu par condensation de metaphénylenediamine avec de la formaline en présence d'acide chlorhydrique avec méthylation consécutive. On se servait dans ce but du diméthylsulfate en quantité de 75—80 pour cent du poids du goudron. La méthylation s'obtenait en maintenant la solution au bain-marie pendant une heure.

Pendant les diverses phases du procédé chimique on déterminait colorimétriquement par le molybdate d'ammonium la quantité de silicium contenue dans l'eau. Cette méthode de dosage donne la quantité de silicium totale—sous la forme de l'acide fluorhydrique et sous la forme de l'acide fluorhydrosilicique[4].

On vérifiait également la disparition de H_2SiF_6 de la solution par la réaction de précipitation en solution alcoolique du barium par les chlorides.

Les résultats sont donnés dans le tableau.

Quantité de SiO_2 contenue dans l'eau en mg/l

Eau de départ	Après traitement par F′ et passage d'air	Après filtration à travers un filtre-donneur d'ions OH′
34	16	<0.5
184	112	<0.5

Quand la quantité de SiO_2 dans le filtrat à sa sortie du filtre-donneur en ions OH′ s'élevait jusqu'à 0.5 mg/l, c'est-à-dire, quand commençait à avoir lieu un libre passage de SiO_2 dans le filtrat par suite de «l'épuisement» de la substance active qui perdait ses facultés de donneur d'ions OH′, on arrêtait la filtration de l'eau contenant SiO_2 et on procédait à une régénération du filtre en le faisant traverser par une solution à 5 pour cent de NaOH ou de Na_2CO_3.

On obtenait ainsi le remplacement inverse de l'ion SiF_6'' par OH′, et l'acide fluorhydrique retenu dans le filtre en était éliminé sous la forme de Na_2SiF_6. L'eau de régénération était rassemblée à sa sortie du filtre, et on y procédait à un dosage de SiO_2. Il s'est montré ainsi que la quantité d'acide fluorhydrique, contenu dans les eaux de régénération, est égale à la quantité retenue par le filtre pendant la filtration de l'eau à épurer, ce qui indique que la régénération du filtre est complète. Les expériences consécutives montrèrent qu'après régénération du filtre-donneur d'ions OH′, ce dernier a pleinement conservé sa capacité de retenir SiF''_6 contenu dans l'eau.

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GEOLOGY

FREQUENCY DISTRIBUTION OF POROSITY COEFFICIENTS AND EPIGENETIC PROCESSES IN SPIRIFEROUS LAYERS IN THE OIL-BEARING REGION OF BUGURUSLAN

By A. B. VISTELIUS

(Communicated by F. P. Savarensky, Member of the Academy, 2. II. 1945)

1. While engaged in the investigation of spiriferous ($P_2^{kaz_1}$) carbonate deposits in the oil-bearing region of Buguruslan in 1943, the author had to solve the following problems:

a) to find a quantitative expression for the results of epigenetic processes in spiriferous layers, which would permit to compile an isoline map of their distribution, and

b) to establish the relation between the processes of cementation and leaching-out in the stratum investigated.

The solution was reached by having recourse to mathematical statistics as described below.

2. As is known from the theory of probability (1), the expression

$$\varphi(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{i\theta x - \frac{\theta^2}{2!}P_2 + i\frac{\theta^3}{3!}P_3 + \frac{\theta^4}{4!}P_4 + \cdots}\, d\theta$$

presents a function of the distribution of probabilities, where P_2, P_3, ... are semi-invariants of complex deviation, each of which is a sum of semi-invariants of individual deviations P_{qr}. The individual deviations P_{qr} are functions of the determinants composed from mathematical expectations of the k-th powers of the r-th deviation.

If all semi-invariants of individual deviations are quantities of the same order of smallness and the number of these deviations is great, then the semi-invariants of a complex deviation P_i diminish rapidly as the number i grows. In other words, P_{i+1} under these conditions are much smaller than P_i, so that the semi-invariants with values of i larger than 2 can be neglected.

Things are different if all quantities P_{qr} are not of the same order of smallness, but some have become considerably larger because of the development of certain processes in a definite direction. Then the semi-invariants P_{i+1}, P_{i+2} ... are not necessarily small in comparison with P_i, and we shall have distributions departing from normal.

3. After these preliminary remarks we turn to our problems to discover the following. Whereas during the period of sedimentation all factors influencing the property investigated are of the same order of smallness, after sedimentation the influence of secondary factors on the sediment grows sharply. Accordingly, individual deviations connected with the epigenetic processes will suffer an in-

crease, the relation between the invariants, necessary for the normal law to be observed, will be broken, and we shall have a distribution type affected by secondary processes.

From the above-said it is obvious that the problem proposed can be solved by means of the following simple method.

We have to obtain parameters of frequency distribution enjoying such a property as would reflect the secondary processes influenced by a change in the relation between the semi-invariants. Mapping the distribution of these parameters, it will be possible by means of isolines to separate the regions of development of the secondary processes. Comparison of the obtained parameters with those of normal distribution will furnish data for decisions concerning the relation between the processes of cementation and leaching.

4. As a characteristic property reflecting secondary processes the porosity of the deposits was chosen. In fact:

a) In the vertical sections of the stratum investigated the porosity had been found to follow a periodical law. Thus, in taking samples at regular intervals one could expect that the selection would in general be a random one. Besides, if we refrain from using average values, it will be all the same to us whether the samples taken belong to one, or to several periods.

b) The porosity is very responsive to secondary changes in the rocks ([4]).

c) At the disposal of the author were several hundred determinations of the constant chosen. They had all been made under identical laboratory conditions, thus furnishing a sufficiently homogeneous material.

The whole region was divided into eight districts, and in each the frequency distribution was studied. The data obtained in this investigation are given in Table 1.

Table 1

District	N	α_1	ι	σ_{α_1}	σ_ι
Buguruslan-Sultangul-Brenchaninovka . .	138	—0.623	+0.382	0.208	0.400
Pilughin	94	—1.048	—0.527	0.252	0.505
Sosnovka	70	+0.037	—0.972	0.293	0.587
Elatom	23	—0.756	+3.508	0.551	1.022
Amanak	175	+0.148	—0.837	0.185	0.360
Zhuravlevka	194	+0.204	—0.975	0.111	0.279
Northern part of Kalinov-Novo-Stepanov .	122	—0.014	—0.006	0.222	0.453
Southern part of Novo-Stepanov	162	—0.238	—0.921	0.192	0.385

After the parameters characterizing the distribution had been obtained, the most typical of them—asymmetry α and curtosis ι—were marked off in the centres of the investigated regions, and through the points thus plotted isopleths([3]) were drawn. Fig. 1 shows the distribution of asymmetry isopleths, and Fig. 2 shows the position of curtosis isopleths*.

As may be seen from these figures, the distribution of epigenetic processes over the area of the region is far from being uniform. They appear to have no influence in the south-eastern districts, while there may be observed an unmistakable stretching-out in the form of a zone towards North-West from the region of oil deposits, where the asymmetry is a maximum and the curtosis is negative. According to the value of curtosis, the whole region can be subdivided into two parts: negative curtosis in the West and positive in the East.

Let us now turn to the interpretation of the observed distribution. The values of asymmetry indicate the relative distribution of porosity departures from average values. The positive values of symmetry correspond to the more porous deposits; the negative values, to less porous. So the increased values of α_1 in the region of

* These isopleth maps are of purely illustrative character, because in plotting them the values of standard deviations were not taken into account.

oil deposits may be interpreted to mean that here the rocks are more often porous than dense, as compared to the normal distribution. Outside of the zone of development of the secondary processes the relation is reverse. The value of curtosis reflects the ratio between dense and porous rocks. Where the curtosis has high positive values, the rocks are uniformly porous. Such is the case in the eastern part of the region. The differentiation of the deposits into dense and porous areas is marked by negative values of ε. This distribution prevails throughout the zone

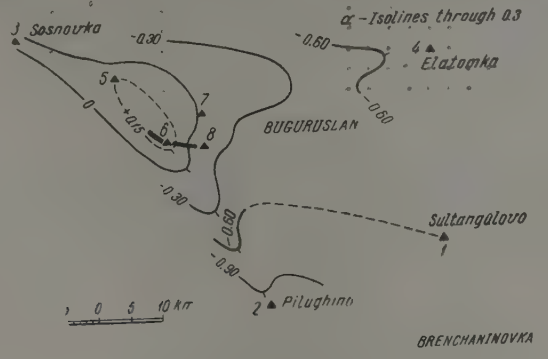

Fig. 1

of secondary changes, widening gradually towards the North-West. Here the deposits have differentiated into separate small areas of dense and porous rocks.

To sum up, we can state that in the western part of the region there may be distinguished a zone of rocks that have undergone secondary modifications, and

Fig. 2

it is here that oil deposits are located. The result of the operation of epigenetic processes in this zone has been a local differentiation into dense and porous portions.

5. The above conclusions were checked qualitatively by a microscopic study of preparations from the deposits investigated, and by inquiring into the degree of mineralization of their waters.

Inquiry into their mineralogy revealed a development of poikilitic structures and an abundance of secondary calcite, native sulphur, sometimes of gypsum and phosphates in the North-West of the region, whereas in the East these minerals occur in sharply reduced quantities. The previous investigators of this region had reported a similar picture (²).

Table 2

District	Well No.	Mineralization per hundred	Specific weight
Buguruslan	22-K	517.878	1.1126
Novo-Stepanovka . . .	322-R	514.016	1.1107
Zhuravlevka	320-R	370.321	1.0810
Kalinovka	327-R	264.020	1.0563
Amanak	461-R	97.012	1,0232
Sosnovka	1-R	21.616	1.0055

The composition of the waters* in the order from East to West is given in Table 2.

From this data it appears that water, as regards its possible effect on the deposits, is increasingly active from South-East to North-West.

It will be obvious that mineralogical and gepchemical evidence corroborate our conclusions.

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GEOLOGY

NATIVE COPPER IN THE LAVA FIELD OF THE SIBERIAN PLATFORM

By G. MOÖR

(Communicated by V. A. Obruchev, Member of the Academy, 15. X. 1944)

In one of his earlier papers (unpublished) the author has shown that within the Siberian platform is located a vast lava field, composed of sheets of basic lavas—effusive representatives of the trap complex. In so far as its composition and structure are concerned, this lava field may be compared to the largest lava areas of the Earth (the Dekan tableland in the British Indias, the Oregon plateau in the basin of the Columbia River in the USA, the lava field of the Karroo formation in South Africa, the basalt fields in the basin of the Parana in South America, *etc.*). The territory occupied by the lava sheets in the North of Central Siberia attains about 300 000—350 000 km²; it is confined to the basin of the right-hand tributaries of the middle course of the Nizhnaya Tunguska River (Kochechuma River with its tributaries, Vivi, Tutonchana and other rivers), to the middle course of the Kotui River, the upper course of the Kureika and Kheta, as well as to the divide elevations between the river systems listed above. The total thickness of the lava in the central parts of the lava field exceeds 1000 m, attaining 250—350 m in the marginal parts.

Until recently the lava field of the Siberian platform was considered only as a promising one with regard to findings of iceland spar, numerous accumulations of the latter being known within the field; in a number of cases these were of high economic value. As a result of investigations carried out in the recent years, however, it seems to be likely to find within the area of the lava sheets, in addition to iceland spar, also accumulations of native copper, which would certainly be of essential importance for evaluating the economic prospects of the lava field.

Records of findings of fragments of native copper within the lava field of the Siberian trap area were mostly spread in the form of personal information; although in some cases they penetrated into literature. Thus, Vysotzky[1] and somewhat later Popov[2] and Riabukhin [3] reported of findings of detached fragments of native copper ores from various points of the northern parts of the Siberian platform. In all cases the points of finding were indicated without sufficient precision, it is true, and, which is particularly important, the ore specimens did never happen to fall in the hands of specialists. Therefore it was not possible to settle the question as to whether this type of mineralization was present within the lava field; it is rather probable that sulphide ores of copper, which do not infrequently occur within the trap province, might be mistaken for native copper.

As is known, native copper occurs rather frequently in lavas of a basic composition. Copper-bearing occurrences of this type have been discovered within the

North-Atlantic basalt province (Farroer Islands, Greenland, Iceland, Scotland), in Brazil and Uruguay, in the trap areas of the North-East of the USA and Canada (New-Jersey, Connecticut, Michigan, Wisconsin), at the Alaska and in north-western Canada (in particular in the basin of the Copper River), on the Komandor Islands (USSR), and in a number of other places. Of particular interest in this respect are the areas of Great Lakes in USA, where the famous deposits of native copper are located, which play a prominent rôle in the production of copper ores.

At all the points listed above the accumulations of copper are associated with basic lavas that have suffered intense zeolitization; the formation of zeolites is related to post-volcanic processes in the sheets; in other words, it was due to the action upon the lava formations (which had no time to solidify as yet) of vapour and gases, as well as of thermal solutions accumulated within the bulk magma and representing the last stages of its evolution.

Deposits of native copper belonging to this type have in the USA a regional development. In particular, they have been recorded from Wisconsin, Michigan and Minnesota, as well as from the areas of Canada adjoining these states; copper occurrences are traceable there over a distance of 500 km; the thickness of the lava series amounts to about 5000 m. Native copper is subordinate to old (pre-Cambrian) lava formations of the Keweenawan. The thickness of individual sheets varies within a wide range, attaining in some cases according to Broderick([3]) 170 m*, in a few cases a still greater thickness being recorded. Copper occurs in amygdaloid rocks (mandelstones), where it fills amygdales jointly with calcite, chlorite, epidote, quartz, prehnite, datolite and zeolites, among which analcime and laumontite are worthy of mention (in so far as the extent of their distribution is concerned).

The most frequently observed are sheets 10—12 m thick, while the zones of mandelstones, in other words, the copper-bearing parts of the sheets proper, have a thickness of 2—3 m; this rather closely approaches to the corresponding figures for lava sheets and the amygdaloidal zones accompanying them in the effusive members of the formation of Siberian traps. The average copper content of the richest areas of amygdaloidal lavas of the Keweenawan makes up about 0.88 per cent. In a few cases large ruggets of copper have been encountered, the weight of which is some hundred tons. These deposits play a great part in the production of copper ores in the USA: since the beginning of working (the seventies of the last century) up to 1940, over 4 million tons of copper have been mined here.

Until recently no safely established findings of native copper ores have been known within the lava field of the Siberian platform. It is true, as a positive indication in this connexion may serve a certain enrichment in copper of the traps of the lava sheets from the basin of the Lower Tunguska, recorded by Tkachenko ([4]). In particular, in the amygdaloidal lavas exposed near the mouth of the Golokit River (left-hand tributary of the Kochechuma, discharging into the latter somewhat upstream of the mouth of the Tembencha River) copper content attains 0.08 per cent; but the sulphide ores, an admixture of which might account for the high copper content, were lacking in the specimen analysed. It should be noted that the figure quoted by Tkachenko considerably exceeds the average copper content of the Keweenawan traps, which varies, according to data by Grout ([6]), from 0.02 to 0.03 per cent; it is also considerably higher than the corresponding figures cited for the lava sheets of the same region by Broderick and Hohl ([7]), according to whom the copper content does on the average not exceed 0.011 per cent. It is a matter for regret that copper was not determined in any of the earlier analyses of amygdaloidal diabases and mandelstones from the formation of Siberian traps; it is not possible therefore to verify whether the increased copper content in the lavas of the Siberian platform has a regional character.

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* These figures refer, as it would seem, to depressions of the ancient relief filled up with lava effusions, since in this case the thicknesses considerably diverge from the average world-values as cited by Wolf ([5]).

In the course of itinerary survey made in the basin of the upper course of the Kotuikan River in summer 1943, the present author obtained new data concerning native copper associated with the effusive members of the trap complex of Siberia. There, among pebble material of the Tomuktasi River, a left-hand tributary of the Kotuikan in its upper course, in a fragment of brownish mandelstone small grains of native copper were discovered. The copper shows a characteristic copper-red colour, low hardness (it is easily scratched with a knife), and is accompanied by oxide compounds dark-green in colour. It should be noted that both the geological situation and the mineral association in which native copper has been encountered in the lavas of the Siberian platform, are characteristic of its accumulations within those lava fields where this type of mineralization is of economic importance (the USA, Canada). One is, therefore, led to suppose that the upper horizons of the lava formation within which native copper has been found, are promising with regard to economic accumulations.

It should be borne in mind, however, that at the time being the finding is of purely mineralogical interest. Now the aim of geological work is to study the processes responsible for the accumulation in the lavas of native copper, and to discover economic deposits on the basis of the data obtained.

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PETROGRAPHY

ANALCIME ROCKS IN THE LITTLE CAUCASUS (AZERBAIJAN SSR)

By A. N. SOLOVKIN

(Communicated by D. S. Beliankin, Member of the Academy, 29. IV. 1945)

In Transcaucasia teschenites and rocks similar to them were described from Georgia by Beliankin (1), Gerassimov (2) and others; and from Azerbaijan, in the Lenkoran region, by Mikhailov (3), Solovkin (4), and others.

In the Little Caucasus such rocks have not been known, which fact impaired the knowledge of the position of the teschenite facies in the folded zones of Transcaucasia.

The investigations carried out by the author during a few recent years in the regions of the eastern foothills of the Little Caucasus revealed a number of interesting data. In the zone of development of the dislocated series of the Upper Cretaceous, composing the border ranges of the eastern and north-eastern foothills of the Little Caucasus, a considerable number of exposures of peculiar intrusions of the basic type have been discovered; in their composition had originally been established differentiates of basic magma, from gabbro to dolerite. As a result of further petrological studies, among these hypabyssal intrusions a second branch of differentiates was established, of alkaline-basic composition, including a more or less typical analcime basalt and gabbro-teschenite. The exposures of the rock named have been mapped chiefly for the eastern marginal part of High (Nagorny) Karabakh, near the villages of Mirishen, Gatzy, Kagarza, Abdal, Takyr, Nakhichevanik, and at some other places.

The geological conditions of these intrusions have very much in common with the intrusion of teschenites of the Tioneti region described by Gerassimov(2). Just like in the latter, the igneous rocks occur here in the form of a sill amidst carbonate Upper Cretaceous rocks. In both regions the deposits of Albian-Senonian age are broken through. In particular, in the region here described the intrusion is located chiefly among marls and limestones of a series of Senonian age (5), but occasionally break through the Albian, too. At any rate, the post-Cretaceous age of the intrusion is beyond any possible doubt.

Among the rocks here described the following types can be distinguished according to their petrographic peculiarities: 1) pyroxene-olivine gabbro-teschenite, and 2) pyroxene-analcime basalt.

The first type is characterized by a crystalline granular structure and a mesostatic nature of the more or less decomposed analcime contained at an amount up to 15 per cent and distributed among crystals of labrador, pyroxene and olivine. Natrolite, chlorite, calcite, apatite and magnetite are common.

The second type of rocks is characterized by an intersertal structure and the presence of analcime as rather large idiomorphic phenocrysts, along with phenocrysts, of plagioclase, titanite and augite. The minerals named do not exhaust

the list of all the components of the rock, among which apatite, natrolite and calcite are common.

The intrusive massifs which doubtless reach a considerable size but are poorly exposed here, include also normal alkaline-earth types: olivine gabbros, dolerite, variolitic basalt. The spatial relations of this branch and of the analcime-bearing one cannot be borne out more precisely; it has been noted, however, that normal gabbros are confined to the deepest portions of the intrusive bodies, while the variolitic basalt is connected by distinct mutual transitions to analcime basalt characterizing the peripheral near-contact zones of the massifs.

It is of interest to note that teschenites known in the region of the Talysh Range (Azerbaijan SSR) since 1932 do not constitute any independent formation either, but are accompanied with rocks of the basic alkaline-earth magma: gabbros, gabbro-pyroxenites, *etc.* As to the teschenites of the Yora River (Georgian SSR), the marginal facies of the intrusion is represented, as shown by Gerassimov[2], just as in the region described, by basalt, having at places the habit of mandelstone.

In the Little Caucasus exposures of gabbro and teschenite-basalt intrusions are quite linearly arranged along the eastern foothills; a considerable part of them are disguised by more recent formations, but remarkably enough, the belt described obviously lies on the line connecting the complexes of teschenite intrusions of Talysh and the basin of the Yora River. This fact makes one assume that the intrusions described belong to a single and independent gabbro-teschenite formation characteristic of the eastern part of Transcaucasia. Thus, the view previously advanced by the author as to the common nature of the teschenites of Talysch and of Yora has been definitively proved.

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PALAEONTOLOGY

REMAINS OF A SEA-TURTLE DISCOVERED IN OLIGOCENE DEPOSITS OF THE NEAR-ARAL REGION

By L. I. KHOSATZKY

(Communicated by I. I. Schmalhausen, Member of the Academy, 23. I. 1945)

During the excavation work carried out by a party of the Palaeontological Institute of the Academy of Sciences of the USSR (J. A. Orlov, J. M. Eglon) at the northern shore of the Perovsky Bay (Sea of Aral) in summer 1938, remains of a very big sea-turtle belonging to the *Cheloniidea* order were found among other palaeofaunistic material.

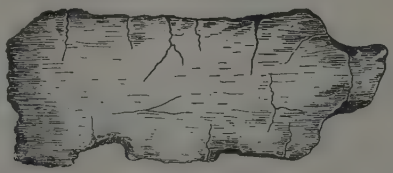

Fig. 1. Costale, left-hand side, viewed from above, 1 : 4

The remains were found some 50 km away from the station Saksaulskaya (Tashkent railway), near the settlements Aghyspe and Ush-Kul, in the Kazakh SSR.

Strata of rocks belonging to the Upper Oligocene, and partly to the Middle Oligocene(1) are exposed there by the nearly vertical fault of the high littoral slope of mountain Aghyspe.

The finding consists of a few unconnected bones, two costal plates of the carapax, one of them nearly intact and another fragmentary, two connected marginal plates of the carapax, and a fragment of the distal part of the coracoideum. Besides these fragments of the skeleton there was found a bone of a very poor state of preservation, which seems to belong to the carpalia. All these bones are strongly fossilized and fragile, especially brittle along the edge.

The material is kept at the Palaeontological Institute of the Academy of Sciences of the USSR.

The total length of the costal plate best preserved measured together with the prominent part of the rib (Fig. 1) makes up 240 mm, its width at the proximal end is 93 mm, at the distal one, 9 mm. The width of the rib at the very edge of the distal part of the plate equals 43 mm. The second costal plate is represented by a fragment of its proximal part whose anterior edge is 115 mm wide; the total length of the fragment measures 180 mm.

The length of the external edges of the two marginal plates is 36 and 34 mm, respectively.

The length of the seam between the two plates (at their external surface) equals 43 mm. The maximum width of the fragment of the coracoideum available is 90 mm (Fig. 2). Judging by the dimensions of the separate bones it may be supposed that this oligocene turtle might well bear comparison with the biggest representatives of the recent *Cheloniidea*.

According to the outlines of the coracoideum and the shape of the plates of the carapax, the turtle here described should be considered as a representative of the *Cheloniidae* family and referred to the genus *Chelonia*, being ranked as a new independent species, *Ch. aralis* sp. n.; this, however, stands in need for additional evidence to be supplied by further more complete findings.

Fig. 2. Coracoideum. Fragment of distal part, 2 : 3

But at the time being the following separate diagnostical characters of this form can already be set forth: concrescence of costal elements with overlying costal plates, complete: ends of costae protruding beyond the edges of the distal parts of the respective costal plates by more than a seventh fraction of their total length; average width of costal plates exceeding the width of corresponding costal elements about 2.5—3 times; marginal plates short, their height at the external surface exceeding the length of the free ends of these plates; caracoidea broader at distal ends and considerably narrower towards their proximal parts.

The age of the sea-turtle should be put down to the Middle Oligocene. At any rate the beds composing mount Aghyspe, where the remains have been discovered, may be referred, in point of their synchronization with the rest of Middle Asiatic subdivisions of Palaeogene formations, to the Turgai indricotherial beds containing terrestrial fauna referable to the close of the Middle, or the very beginning of the Upper Oligocene.

The discovery in the proximity of the Aralian littoral of remains of big aquatic turtles which, as well-known for other representatives of this order, inhabit vast marine reservoirs*, should be interpreted as evidence of a vast and deep sea having existed there till the beginning of the Upper Oligocene at least. This reservoir should have suffered reduction during the Upper Oligocene, which may be inferred from the occurrence of various terrestrial fauna, including also terrestrial turtoises (*Testudo aralensis* Khos. sp. n., in litt.) in the deposits of that time exposed in the vicinity of the same Aghyspe mountain.

Before the finding of the remains of the species here described, fossil sea-turtles had been reported for this country from the Lower Oligocene, found near

* Along with bones of the marine turtle remains of big cetaceans have also been found in the same beds.

the town of Ordjonikidze (North Ossetian ASSR) (*viz.* the species *Chelonia caucasica* Riab.(2)) and from the Jurassic deposits of the mountain system Karatau in Southern Kazakhstan (*viz.* the species *Jaxartemys longicauda* Riab. gen. n., sp. p., in litt.).

These remains are, however, represented by imprints of young specimens in shale-plates only; the discovery of bones of a big marine turtle of the species here described is therefore of great interest, as an important contribution to the knowledge of the fossil fauna of the vertebrates of this country.

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HISTOLOGY

CORRELATION BETWEEN THE SKIN TEXTURE IN KARAKUL LAMBS OF ROBUST CONSTITUTION AND VARIOUS TYPES OF THE KARAKUL CURL

By N. V. POPOVA

(Communicated by A. A. Zavarzin, Member of the Academy, 8. XII. 1944)

The object of the present study was to establish a regular correlation between the skin texture of Karakul lambs of robust constitution and the perfect or vicious types of the karakul curl. With this aim in view skin samples of Karakul lambs were taken at a sheep pedigree farm «Kara-Kum»; the age of the lambs ranged from 1 to 3 days.

The specimens were sorted out for examination during the lambing period. The furs were sampled in a rigorously similar way, the fragments of skin being cut out from the sacral region and fixed with 10 per cent formalin. After being embedded in colloidin-paraffin, the sample was cut perpendicular to the longitudinal axis of the fur, along the length of the hair-roots in the thickness of the skin.

The first part of our work consisted in examining the general aspect of the skin structure corresponding to the two extreme types of the curl of the Karakul fur, *viz.* to the best type of curl, known by the local name «valek», and to a total lack of any clearly expressed curl, considered as waster, and termed the «felt», or felt-like fur.

The typical picture offered by a skin texture corresponding to the perfect curl-type, «valek», can be characterized in the following way. The hair-roots are arranged in the skin layer strictly parallel to one another. All the bulbs are situated at a similar depth and possess well developed nipples of a cone-like shape.

Besides, the hair-roots form two typical bends, one at the point of the hair-neck, another somewhat above the neck, in a direction opposite to the former. Owing to these two kinks, the roots represent highly elastic structures, sigmoid in shape. The main roots are similar in shape, with well developed bulbs. The arithmetical mean of the cross-section of the bulbs, as expressed in points of a micrometer eye-piece, is twice that recorded in the case of felt-like fur. The same is true of the thickness of the skin and the depth of the hair-bulbs (see table).

In lambs producing felt-like fur the hair-roots are found to be much thinner; they are highly unequal in size and show no double kink described above, which is evidence of the elasticity of the hairs. One may not infrequently observe hair-roots which stand upright as sticks, with poorly developed bulbs of an oblong shape. Within the thickness of the skin the hair-roots are not always arranged at the same level.

Curl-type	Arithmetical mean in points of micrometer eye-piece, in μ		
	cross-section of hair-bulbs	depth of situation of hair-bulbs	depth of skin
Perfect curl, «valek»	1.10	9.2	13.15
Curl bean-like	1.70	9.1	13.3
Felt-like deformation	0.82	8.0	11.4

Now it seemed of interest to trace the variations in the quality of the hair in the case of the numerous deviations from the curl-type «perfect valek», which lead to its deterioration, *viz.* «flat valek», «valek with an excessively broad seam», «valek partly rib-like», «open rib-like valek», *etc.* All the deviations in the shape of «valek» listed above have one feature in common, *viz.* in all of them the hair fibre becomes thinner and less elastic, the bend of the hair near the neck vanishes nearly altogether, and the second bend, above the neck, is also somewhat obliterated. The bulbs are elongated, the nipple of the hair is less expressed. The regularities in the hair arrangement, *i. e.* their parallel arrangement and the similar level of growth, are also less uniform than in the case of a perfect, normal «valek». The general histological picture is thus somewhat different; it seems to be intermediate between that observed in the case of a normal «valek» curl, and that associated with the «felt-like» deformation.

In the case of the «bean-like» modification of the Karakul curl the histological picture observed approaches that described for the perfect, «normal valek». The roots of the hair are thicker, however, and so are the bulbs (Table 1); the second kink is situated somewhat higher and is less clearly pronounced, which impairs the elasticity of the hair fibre. A preliminary analysis of the skin samples taken for the cases of vicious curls has shown the quality of the hair to occupy a position intermediate between that of a first-rate type of the curl, and the felt-like deformation.

Discussion. The book «The Karakul furs»([1]) contains a brief description of the skin of Karakul lambs corresponding to three types of curl, *viz.* valek combined with bean-like, annular and pea-like, and, lastly, to the felt-like deformation. There is also a histological analysis of the fur of Karakul lambs at the age of three days. Our data do partly coincide with the description of the hair texture in the case of the valek and bean-like curl, and also in the case of the felt-like deformation. We studied the skin of Karakul lambs not merely for the «valek» type of the curl, but for its most perfect form, «normal valek», and also for all kinds of deviations from and gradual transitions to less perfect forms. In this way we have succeeded in tracing the transition in the skin texture taking place mainly at the quality of the hair, from the «normal valek» down to the felt-like deformation. It should be pointed out that all our observations refer to a single constitution of sheep, *viz* the robust one.

The examination of a large number of sections prepared with skin samples taken from first-rate lambs gives a clear idea of its uniform texture, which differs considerably not only from the one observed in the case of «valek» showing a tendency to become vicious altogether, but even from the texture of skin in the case of «bean-like» curls. As a matter of fact, in the literature devoted to this subject, the texture of the skin in the case of «valek» is described as being analogous to that of the «bean-like» modification.

On account of the uniform texture of the skin observed in the case of a perfect type of the curl, we believe that under adequate conditions the histological method

of skin examination might be applied to breeding work. It would enable the sheep-breeder to sort out biologically valuable animals.

The deterioration of a good «valek» curl and its transition to the felt-like deformation is accompanied by a thinning-out of the hair-fibre (see table), which at the same time becomes as it were coarser. The regular parallel arrangement of the hair-roots in the thickness of the skin gives place to a rather irregular distribution of the hairs. The texture of the skin in the case of felt-like deformation is thus found to approach that observed in the case of adult ewes of delicate constitution. It would be of interest to find out whether the felt-like deformation of the curl is hereditarily transmitted by ewes of delicate constitution, along with the properties of their skin.

From the data available on the histology of the wool of Karakul lambs corresponding to the various types of the curl it is known that: 1) the formation of a perfect curl depends upon a definite cross-section of the core, and 2) the plasticity and elasticity of the wool of Karakul sheep is due to a poor development of the core and an abundant development of the cortical layer of the fibre. It may therefore be assumed that the alteration of the quality of the hair in the case of transition from a perfect curl to a felt-like deformation, or in the case of vicious curl-types (crest-like, pea-like curl, «corkscrew», *etc.*) is connected with different degrees of thickening of the core of the hair-roots. This makes them loose their elasticity, the typical double «kink» of their root, and prevents them to form a perfect curly fur. This variation in the quality of the hairs (fleece) is underlain both by hereditary constitutional factors and by factors of the surrounding medium, food and other conditions to which the ewes are exposed during the period of their pregnancy.

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HISTOLOGY

AGE VARIATION OF THE SKIN OF KARAKUL SHEEP

By V. A. SHOLOKHOV

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It is well known that during the first 30 days of their post-embryonic life the skin of Karakul sheep undergoes a number of changes which lead to a gradual increase in its total thickness, as well as to an increase of almost all its histological elements (1). For Karakul-sheep breeders, engaged along with breeding and hybridization also in the study of the pattern, composition and texture of the wool covering, it is of paramount importance to correlate all these variations with the morphological processes that take place in the skin of Karakul sheep in the course of their subsequent development.

The work here reported was carried out with the purpose to follow up the changes which occur in the histological structure of the skin at the age from 6 months to 1 year, 2 years and older. This subject has been given but little attention as yet; and we know in literature nothing except fragmentary information by Pfeifer (1), who points out that the thickness of the skin of Karakul sheep does but slightly increase with age. Herre and Rabes (2) speak also of insignificant changes in the skin of Karakul during the first year of life; these consist in a thickening of the intermediate layer of the cutis, in an increase in the sweat-glands; the total thickness of the skin also increases in proportion to the total increase in the length of the hairs.

For the present investigation use was made of material collected at the pedigree farm «Kara-Kum», Bukhara province, Uzbekistan. Skin samples were taken from pure-bred rams of robust constitution, at the age of 6 months, 1 year, and above 2 years of age; when young lambs, all of them showed a perfect type of curl known by the name «valek». Fragments of the skin, 1—1.5 cm^2 in size, were cut out with the aid of scissors from the sacral region, and immediately fixed with 10 per cent formalin.

Histological sections manufactured from celloidin-paraffin blocks were stained with Carazzi's haematoxylin with subsequent after-staining with water solution of eosin and with orsein, the latter treatment being applied with the aim to bear out the elastic fibres.

At the age of 6 months, 1 year and above 2 years the skin of Karakul rams has been found to preserve all the topographical peculiarities of texture described for the skin of male lambs during the first month after birth. According to Herre and Rabes, the subdivision of the mesenchymal part of the skin, as suggested by Bonnet for domestic mammals in general (subepithelial, intermediate, reticular and subcutis, all these layers being not sharply delimited from one another), may well be applied to the skin of Karakul sheep, as well. Our own observations make us comply with this opinion, with regard to both young male lambs and adult rams.

According to our data, the skin of rams has completed its total thickening towards the 6th month of growth of the animal. We failed to observe any changes in the thickness of skin in animals at the age of 1 year and in those above 2 years old.

As the skin stops to grow, however, the subsequent development and differentiation of its saparate components take on a more sharply pronounced character.

The epidermis of the skin, which by the end of the 1st day of post-embryonic life shows an average thickness of 19.2 μ, attains an average thickness of 31.0 μ towards the 6th month of development. At the age of 6 months or 1 year it still remains but poorly pigmented, the amount of pigment being somewhat larger within such areas as surround the hair sheath. During this period the epidermis still consists of 3—5 layers of cells of the Malpighian layer, in which also pigment grains of the corneous layer are to be found. Both the granular and the lustrous layer are lacking, and a layer of granular cells may but infrequently be observed. In the case of anomalous development only, consisting in a vicious position of the hair, which had been observed in one of our sections, was it possible to record a well pronounced granular layer. At the age of above 2 years the epidermis undergoes a number of sharp alterations: the number of cell-layers increases up to 12, the amount of pigment grains also increases sharply; they fill up all the lower cells of the Malpighian layer. The granular layer becomes sharply conspicuous at the same time. The table adjoined gives an idea of the variation in the average thickness of the epidermis.

Changes in the Thickness of Epidermis

Age	Average thickness of epidermis, in η
6 months	31.29
1 year	32.80
Above 2 years . .	42.40

The mesenchymal portion of the skin which at the age of 1 day shows an average thickness of 2030 μ attains a thickness of 3448 μ towards the 6th month. At the age of 1 year and after 2 years the skin ceases to increase in thickness. A structural peculiarity of the skin, which remains common to all animals during the period investigated (6 months, 1 year and above 2 years), is the presence of collagenous fibres of a much coarser structure, and of a much smaller number of cellular elements of the connective tissue fibroblasts; this is an important distinctive feature which makes this material different from that of earlier stages. The reduced number of fibroblasts in the skin of rams at this age should probably be accounted for by processes of differentiation that take place in the skin, and by the gradual proliferation of the cells of the connective tissue, which results in a slowing down of the growth processes of the skin. But the subepithelial layer invariably shows a larger number of fibroblasts than the underlying layers do.

The bulbs of the roots of the primary raw of hairs, which at the 1st day after birth show an average thickness of 116.8 μ, attain towards the age of 6 months an average thickness of 191.2 μ, but during their subsequent development they cease to increase in thickness. Like in young lambs, the roots of the primary hairs of the skin preserve throughout the subsequent development stages their parallel arrangement; their bulbs are situated at about the same depth. This should be interpreted to mean that their peculiar arrangement in the skin of such lambs as are endowed with the highly valuable character of the perfect curl, «valek», is

preserved in adult rams, sires, which are sorted out for breeding purposes, in the first place according to the quality of the pattern of their curls.

The roots of the primary hairs in rams at the age of 6 months preserve the peculiar shape characterizing new-born and young lambs. The hair bulb is flask-like in shape, with a triangular nipple. At this age of the animals it is already possible to observe the typical sigmoidal bend of the hair, brought about by a kink at the neck of the hair, and by an opposite kink at a higher part of the hair. Like at the early age, the roots of the hairs of 6-month-old animals are liberally supplied with black or dark-brown pigment. In the skin of 1-year-old rams, however, along with hair-roots of the structure described above, one happens to observe also such roots as show no sigmoidal kink, and are destitute of pigment. The average thickness of the bulbs of both pigmented and pigmentless hairs is about the same. The roots of pigmentless hairs do not form any appreciable neck of the bulb, as pigmented hairs, but instead their bulbs pass directly into the root of the hair. The nipple of such hairs is rather elongated in shape, differing from those of pigmented hairs. The roots of the hairs of this type are somewhat bent at different points, or stand quite erect at the surface of the skin. Such pigmentless hairs make up the largest portion of the primary raw in the skin of rams after they are two years of age.

Along with hair-roots of the primary raw, which are situated at a larger depth of the skin, the upper layers of the skin of 2-year-old and still older animals contain also thin roots of fluffy hairs, endowed with all the characters of the roots of pigmentless hairs described above.

The depth of insertion of the roots of the primary hair raw, which at the first day of life measures about 1192 μ, is found to increase to 2370 μ by the 6th month; in the course of the subsequent development of adult animals these figures show no appreciable variation.

The appearance of pigmentless hairs may possibly be accounted for by their formation anew, rather than by a loss of pigment by the hairs showing a sigmoid kink.

This assumption is supported by the absence of any transitionary forms between pigmented and pigmentless hairs, the presence of fine pigmentless hairs in animals at the age of over 2 years; the fact that these two types of hair-roots are invariably observed in newborn grey lambs also speaks in favour of this assumption.

Up to two years the shape of the sebaceous glands undergoes no apparent changes, preserving all the features peculiar to those of young animals. After two years, however, the bag-like simple sebaceous glands start branching. At birth their average length is 287.5 μ; towards the age of 6 months it shows an increase up to 362.5 μ, and they cease to grow after the animals have attained the age of one year.

In animals half-a-year, one-year or two-year-old, the shape and size of sweat glands varies greatly. The secretory part of the gland is usually found to reach with its lower end the level of insertion of the hair bulb, penetrating occasionally even to a deeper layer of the reticular tissue. In the skin of rams at the age of 6 months the secretory parts of the sweat glands are the shape of rather sinuous tubules; in rams after one year of age this sinuosity attains a still higher degree. After two years the secretory sections of the sweat glands have developed so many curves that on a section one never succeeds to observe a more or less extensive segment of the duct, only separate dissected fragments being scattered all over the field of vision.

Conclusions. 1. In Karakul rams selected for breeding purposes at an early age according to the perfect pattern of their curl, known by the name of «valek», the total increase in skin thickness is complete towards the age of 6 months.

2. After the age of 6 months, when the thickness of the skin shows no further increase, the development of the separate histological components of the skin shows a much stronger degree of differentiation.

3. The epidermis undergoes further development, its thickness increasing at the expense of multiplied layers of cells of the Malpighian layer; towards the age of two years it assumes a well-expressed granular structure. At this age of the animal the Malpighian layer contains more pigment grains than at earlier stages.

4. At the age of 6 months, 1 year and over 2 years the sweat glands continue to increase in size; this increase is accompanied by a further differentiation which consists in an increasing sinuosity of their secretory portions.

5. After the animal has attained the age of 6 months, its sebaceous glands do not increase appreciably; after two years, however, the secretory parts of glands develop numerous branches.

6. At the age of 6 months the hair-roots still conserve all the traits peculiar to young lambs; after one year, however, pigmentless, erect or slightly bent hairs make their appearance among the pigmented roots of the primary hairs. After the age of two years most of the roots of the primary raw are found to be substituted for by roots of this type.

7. The parallel arrangement of the hair-roots, which is typical of the perfect curl pattern known by the name of «valek», is preserved at all stages of the animal's development: it is especially well pronounced in the skin of young lambs during the first days of their life, when the pattern of the curl is also most clearly observable.

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PLANT PHYSIOLOGY

GEOTROPIC REACTION OF LEAVES AND CONTENT OF GROWTH HORMONE IN PLANTS

By L. P. ZHDANOVA

(Communicated by D. N. Prjanischnikow, Member of the Academy, 29. IV. 1945)

According to the modern views, the geotropic reaction of the plant organs depends on an uneven distribution of the growth hormones. Cholodny([1]) and Went([2]) were the first to formulate this statement. In the subsequent studies ([3–9]) the hormone theory of geotropic reaction and of tropism in general was fully corroborated by experiment, and received theoretical foundation.

Du Buy ([10]) proved that the intensity of the geotropic reaction depends on the absolute difference in the content of growth hormones at the upper and lower surfaces of a plant organ. Dolk demonstrated at a still earlier date ([11]) that the geotropic reaction of decapitated coleoptiles of oats was proportional to the quantity of growth hormone administered from outside through the wound surface; this was later on corroborated by Katunsky ([12]).

The literature on the rôle played by growth hormones in the geotropic reaction is very scarce and deals chiefly with the movements of the leaves and petioles when attached to the plant ([13–15]). In most modern reviews this problem has not been discussed at all.

The present article reports certain data on the connexion between the geotropic reaction of the leaves and the content of growth hormones.

The following technique was applied to measure the geotropic reaction: young fully developed leaves were severed from the plant at the base of the petiole and put in a horizontal position on narrow wooden boards into moist chambers, in such a way as to make their greater parts hang unsupported. After 1—3 days the leaf blades, and to a higher degree, the petioles, developed bends which were measured by the angle formed by the horizontal line and the basal part of the petiole. To ensure greater precision the angle of bend was measured in the beginning and at the end of the experiment, the difference serving as an index of the magnitude of the geotropic reaction of the leaves.

It was found from the very first experiments on the geotropic reaction of the leaves which were severed from *Hydrangea*, perilla, mustard and Jerusalem artichoke (variety «Kiev white») that no geotropic reaction took place unless the leaf was oriented with is dorsal surface upwards, as shown in Fig. 1. When the orientation was an opposite one, no geotropic bends whatever were observed.

In a leaf which is severed from the plant and placed horizontally, as well as in cylindrical stem organs under the influence of the force of gravity there occurs redistribution of growth hormones and their accumulation on the lower side. However, stimulation of growth of this side and a resulting geotropic bend are observed only in case the leaf is turned downwards with its ventral side; it is not observed whenever the orientation of the leaf is an opposite one.

This phenomenon evidently depends on different growth reactions of the dorsal and ventral sides of the petiole; this was also supported by the experiments of Mai [13]. In Mai's experiments there was a quicker growth of the ventral side of the petiole than that of the dorsal under the influence of pollen of orchids applied to the surface of the cut of the petiole and serving as a source of growth hormones. One may think that differences in the growth reactions of the two opposite sides of the leaf serve as a basis for the mechanism of the epinastic movements. From this point of view epinasty of the leaves should be attributed to the increase in the content of growth hormones in the plant, and, consequently in the leaf, which results in a more intense growth of the ventral side of the petiole. A number of studies, including the comprehensive investigation by Fischnich [14], point to the

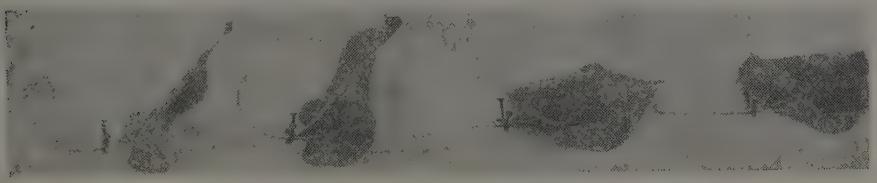

Fig. 1. Geotropic reaction of leaves of Jerusalem artichoke, variety «Kiev white». Left, leaves turned with dorsal side upwards; right, with ventral side upwards

relation between the phenomenon of epinasty and the growth substances content. When the content of growth hormone is reduced, the growth of the ventral side is slowed down and the leaf rises, thus showing hyponasty, for example, in decapitated plants with reduced growth hormone content.

In the series of experiments that followed, we determined the value of the geotropical reaction of the leaves in intact and in previously decapitated plants. We showed earlier [15] that in the green plants the apical bud is the main centre of growth hormone production; the hormones produced there are distributed down the stem and into the leaves. This was proved by decapitation experiments with *Vicia faba*, with tomato (variety «King Humbert») and with Jerusalem artichoke. It appeared from these experiments that the growth hormone content was sharply reduced in decapitated plants, being there only 20—30 per cent of the control.

For the experimerts on the geotropic reaction of the leaves plants of *Hydrangea* were taken, in which the apical bud and all the side buds were removed. In a portion of the plants 2 per cent growth paste (a mixture of laroline with β-indolyl-acetic acid) was applied to the surface of the cut after the removal of the apical bud, as a source of the growth substance. After some 6—20 days following the beginning of the experiment the geotropic reaction of the leaves taken from intact and decapitated plants was determined. The replication of the experiment was a twofold one. The results obtained are presented in the table.

The data contained in the table show that the leaves of decapitated plants, *i. e.* of those whose growth hormone content had been strongly reduced, gave a much slighter reaction to the influence of the gravity force as compared to the leaves of the intact plants or of the leaves of plants which had been decapitated, but received, instead of the apical bud, the growth substance. The following experiment involving plants of similar age but taken at different developmental stages points to the connexion existing between the geotropic reaction of the leaves and the content of growth hormone of the plant. White mustard (long-day plant) and perilla (short-day plant) were taken. The plants were grown under natural diurnal illumination and in a 10-hour-long day. Toward the moment of sampling the mustard grown under long day was at the beginning of fructification; that cultivated under short day has reached the phase of vegetative growth. The growth hormone content in the apical portion of the stem was determined by the diffu-

Geotropic Reaction of Leaves of Decapitated and Intact Plants of Hydrangea

No. of experiment	Number of days from decapitation to cutting off leaves	Condition of the plants								
		Decapitation + + lanoline			Decapitation + + growth paste			Intact plants		
		Bend of the leaves measured in degrees								
		before experiment	after experiment	difference	before experiment	after experiment	difference	before experiment	after experiment	difference
1	10	0	7	7	7	45	38	0	54	54
2	13	3	10	7	11	48	37	5	65	60
3	6	0	21	21	0	31	31	6	41	35
4	12	0	14	14	8	75	67	5	82	77
5	16	7	11	4	0	37	37	0	49	49

sion method after Went, and measured in degrees of the bend of the coleoptiles of oats. In the vegetatively growing plants it was 16°, while in the fruiting plants no growth hormones could be detected.

The perilla plants which were grown under short day conditions attained the fruiting phase while those grown under long day were still in the phase of intense

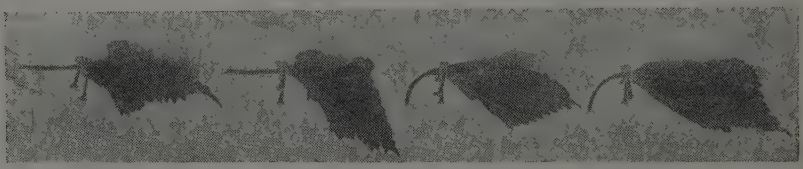

Fig. 2. Geotropic reaction of leaves of perilla. Left, leaves taken from growing plants; right, from fruiting plants

vegetative growth. A sharp difference was observed between the behaviour of the leaves taken from growing plants and that of leaves belonging to plants producing fruits. The geotropic reaction of the leaves from growing plants of mustard was intense: the angle of deflection of the leaf from the horizontal line was equal to 50 and 55°, while the leaves taken from fruiting plants did not react to gravity at all. The leaves of perilla behaved in a similar way. In Fig. 2, left, are shown leaves taken from growing plants of perilla showing a sharp geotropic reaction; on the right half of the same figure are shown leaves of plants at the beginning of the fruiting period, which show no bends whatever.

The following conclusions are to be made from the experiments described above:

1. The geotropic reaction of leaves severed from the plant does not occur unless they are oriented with their dorsal side upwards; it is lacking in case the orientation is an opposite one. This fact may be explained by a certain difference in the anatomical or morphological structure of the two opposite sides of the petiole and a corresponding difference in their reaction to the effect of the growth hormones. Epinasty and hyponasty of the leaves of the plants must probably depend on the concentration of the growth hormone and on the difference in the intensity of the growth processes in the ventral and the dorsal sides of the petiole.

2. The degree of the geotropic reaction of the leaves depends on the content of growth hormones which are synthesized in the apical buds of the stem.

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PLANT PHYSIOLOGY

OVERCOMING NON-CROSSABILITY IN COTTON

By A. S. KASPARJAN

(Communicated by A. A. Richter, Member of the Academy, 4. III. 1945)

Many endeavours have notably been made by cotton-breeders to obtain hybrids between gusa ($2n=26$) and the American forms of cotton cultivated in the USSR ($2n=52$). It seemed highly desirable to impart to the fibre of the valuable American cotton varieties the woolyness of the fibre of the gusa. But aside from being wooly, the gusas possess a number of other valuable characters.

A small number of hybrid plants could be obtained with the aid of Michurin's methods. These hybrids, though nearly sterile, served nevertheless as a source for obtaining a number of forms which proved of practical importance.

Our work has shown that with the aid of polyploidy the breeder becomes able to overcome such important difficulties as non-crossability.

The work was carried on at the Tashkent experiment field of the Institute of Botany and Pedology of the Uzbek Branch of the Academy of Sciences of the USSR in 1941—1942. As material use was made of various forms of gusa possessing definite valuable characters, 7 forms of upland and 3 forms of Egyptian cotton.

In the first line we had to produce tetraploid gusa. The technique of the production of tetraploids was as follows. The seeds were first treated with sulphuric acid in order to destroy lint and then placed into a thermostat to be germinated in a 0.05 per cent aqueous colchicine solution, at a temperature of 25—30° C. A portion of the seeds was subjected to colchicine treatment after having been previously germinated in a thermostat at the same temperature. The duration of the treatment varied from 18 to 50 hours. After 18 hours such seeds as showed the strongest germination were removed from colchicine. Those which have germinated but slightly or did not yet germinate at all were left in the solution for 33 and 50 hours. After colchicine treatment the seeds were rinsed and planted in small paper boxes.

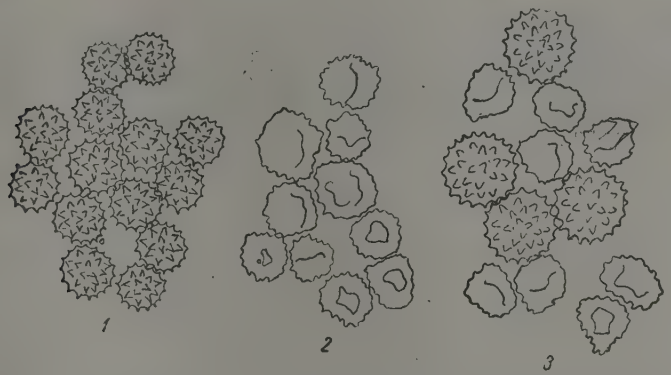
The critical period for the treated seeds was that of the first fourty-five days; during this period their growth was suppressed, the main root died because of the disturbances taking place in it, while the lateral roots did not yet develop. The failure of some investigators in these experiments should be explained by the mass destruction of the treated material at this stage.

We have treated a total of 200 seeds (10 seeds of each sample). There were obtained from them 66 plants possessing duplicated chromosome numbers, and 8 plants which remained unaffected.

The gusas which possessed doubled chromosome complements ($2n=52$) differed from the initial material in a deeper green coloration, in strikingly larger organs, thick stems, solid large leaves, very strong pubescence and somewhat deformed and intensely coloured flowers. The plants were totally sterile. In the

majority of cases the pollen was totally sterile, but, as shown in the figure, some of the form produced a certain amount of viable pollen. The diameter of these viable pollen-grains was about 50 per cent larger than that of the normal diploid ones. The variability of the egg-cells was high and attained 50 per cent. The growing period was more or less increased.

Our results coincide with those of Beasley (1) who had obtained a tetraploid in *Gossypium herbaceum* L., with the only difference that his tetraploid was somewhat fertile. The fibre which developed on the few filled seeds was 1.4 times longer and 1.3 times thicker than in the diploid. The tetraploid *Gossypium arboreum* var. *neglectum* Watt. which was obtained by Stephans (2) had, like in our case, almost sterile pollen and from 40 to 50 per cent of viable egg-cells.



Pollen of *Gossypium arboreum*, 02090: *1*, from a diploid plant; *2*, from a tetraploid plant; *3*, pollen of *Gossypium arboreum* 02451, tetraploid plant

Nearly all the flowers which had opened were used for crossings. Pollination was done without castration. The total number of crosses was 1000 and 2706 seeds resulted from them.

The hybrid nature of the seed obtained is testified by the fact that all the 50 seeds planted in 1942, produced haplo-diploid plants, *i. e.* such plants as possess two chromosome sets of the gusa variety and one chromosome set of the American cotton varieties. Our experiments thus went to demonstrate that the doubling of the chromosome set in the gusas led to a sharply pronounced change in their behaviour with regard of crossability with upland and Egyptian varieties of cotton.

While the tetraploid forms of gusa aquired the ability of crossing with the American cottons, they did, on the other hand, lost the crossability with the original diploid species. About 400 crossings made by us to check this point failed to produce any bolls.

The results obtained by us show once more that polyploidy is an extremely strong factor which causes striking changes in the conditions of crossability of a species.

The behaviour of the 50 hybrid plants studied by us was highly variable. The study of the 20 different combinations at our disposal enabled us to trace a definite regularity in their behaviour. Heterosis was the strongest in the hybrids of gusa, namely of *Gossypium arboreum* L. with Egyptian cottons. They were vigorous and could easily be distinguished among all other forms; their stems were thick and their leaves very large, undulate and strongly lobate. Heterosis was especially strong in the hybrids between *G. arboreum* and *G. barbadense* variety Maarad. These hybrids were twice and even more as tall as the original forms. Crosses between Upland and Gusa showed no such heterosis and in some cases produced even somewhat depressed plants (see table).

Characters	*G. Nanking* × × *G. barbadense*			*G. Nanking* × × *G. hirsutum*	
	F_1	♂	♀	♂	F_1
Height of the plants, cm	155	105	130	127	126
Number of nodes	21	18	23	24	22
Length of internodes, cm	6.5	6	5	5	5
Length of leaves, cm . . .	14	13	9	12.5	11.5
Width of leaves, cm . . .	15.5	14.5	10.5	13	13.5

The features common to all hybrids studied were: dark green coloration, solid leaves with increased relative width, roughness of the plants, their great fragility. Since the hybrids were no complete amphidiploids, *i. e.* contained only one diploid chromosome set belonging to the one of the parental species, their meiosis was very irregular, which resulted in nearly sterile pollen.

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PHYTOPATHOLOGY

A CONTRIBUTION TO THE STUDY OF THE PHYSIOLOGY OF GALLOPING WILT IN COTTON

By P. V. SABUROVA

(Communicated by A. A. Richter, Member of the Academy, 13. V. 1945)

The galloping wilt in cotton, or the black sclerotium root-rot, is caused by the fungus *Sclerotium bataticola* Taub. Externally the disease is expressed in a sudden wilting of the whole plant. Withering and death of the plant are sympthoms characterizing the latest phase of the disease; the beginning of the latter must always be referred to a much earlier date.

The aim of the present study was to disclose the mechanism underlying this disease.

For this purpose use was to be made of data secured from a study of the physiology of the water regime of the affected plant and of the physiology of the fungus grown under artificial conditions, as well as from a study of the toxic effect of the fungus upon the plant. The latter part of the work was conducted on two fungal species, *viz. Sclerotium bataticola* causing galloping wilt, and *Verticillium dahliae* Kleb. causing chronic wilt. The experimental plants belonged to variety 1306. They were grown in a greenhouse in pots. Inoculation was done by introducing the infection into the soil when charging the pots.

Parallel physiological studies of the diseased and sound plants were conducted at different dates. Forcing power of the root, transpiration intensity and water content in the leaves were studied. For the determination of the forcing power of the root the plants were used which served to take records of transpiration. The forcing power of the root was determined with the aid of Sabinin's method (4), after the amount of sap liberated within unit time. The results are presented in Table 1.

Table 1

Amount of Sap Excreted in g per hour

Date	Sound plant		Diseased plant	
	absolute	relative	absolute	relative
10.VII	0.0233	100	0.0168	71.24
1.VIII	0.0723	100	0.0647	89.48

It may be seen from the table that the quantity of sap excreted by diseased plants was smaller than that excreted by the healthy ones. The difference was on the first date 28.76 per cent, and 10.52 per cent, on the second. The decrease in

the forcing power of the root, and of the sucking force of the root which is connected with the former observed in our experiment in diseased plants, was a result of a disturbance of the osmotic force of the root. A reduction in the volume of the root-system, namely of the absorbing zone or, more precisely, of the zone of the root-hairs, consequent to the attack of the fungus, might also have led to a reduction in the sucking and pressing forces of the root.

Transpiration was determined by weighing the pots with the aid of a Beranger balance every third hour, beginning at 8 o'clock a. m. till 4 o'clock in the afternoon.

The experiments were carried on during three days, at two dates. The data on transpiration are presented in Table 2.

Table 2

Transpiration Expressed in g/hour per dm² of Surface

Date	Transpiration intensity			Air temperature, °C	Weather conditions
	sound plant	diseased plant	in % of the healthy		
8.VII	2.67	1.77	66.29	30.4	Cloudless
9.VII	3.66	2.50	68.36	36.6	»
10.VII	4.15	2.47	59.51	36.6	»
30.VII	0.59	0.57	96.61	22.0	Unsteady
31.VII	1.14	1.11	97.37	26.0	Cloudless
1.VIII	1.67	1.52	91.02	28.4	»

When comparing the above figures, we find a difference in the transpiration intensity between the diseased and healthy plants to the disadvantage of the former. The difference recorded on the first date was well-pronounced. Later on this difference has become less conspicuous. It should be pointed out that the transpiration of both diseased and healthy plants is found to be affected in a similar manner by the external conditions; transpiration increases along with the increase in the intensity of the atmospheric factors; with the decrease in them transpiration suffers a parallel reduction. The increased transpiration observed in cloudless sunny days is not deleterious, if water expenditure is fully compensated for by its inflow. If the supply of water is reduced, water deficiency sets in, and the plant suffers withering.

We have observed no noticeable water deficiency in the diseased plants. The determinations showed scarcely any difference between the diseased and healthy plants with regard to their water content. Consequently, water expenditure both in the diseased and in healthy plants was compensated for by the debit. A reduction in water supply to the above-ground parts in the diseased plants leads to a reduced consumption of water, which is brought about by a reduction in transpiration; this is a defence reaction of the diseased organism.

The physiology of the fungi was studied under artificial conditions. The basic nutrient medium had the following composition (pro 1 l of distilled water): glucose, 20 g; peptone, 2.9 g; K_3PO_4, 1.9 g; $Ca_3(PO_4)_2$, 0.9 g; $MgSO_4$, 0.4 g; NaCl, 0.6 g; FeCl, traces.

The fungi were grown at a temperature of 28—30°C during two months. After this period different forms of nitrogen were determined (total protein, ammonia and amide nitrogen) in the fungus and in the medium in which the fungus had been grown; finally, the dry substance was weighed. The data on the weight of the fungi (average from three determinations) are presented in Table 3.

It may be seen from Table 3 that *Sclerotium bataticola* produced within the same period of time nearly twice as much dry matter as *Verticillium dahliae*. The intensity of growth in the former is therefore twice that of the latter. The intense growth of *Sclerotium bataticola* under artificial conditions shows that this fungus

Table 3

Fungus	Dry weight, in g	Nitrogen content in % to the dry weight		
		total	protein	soluble
S. bataticola	0.3663	3.16	1.63	1.53
V. dahliae	0.1898	6.24	3.49	2.75

is more agressive, and that its parasitism is more pronounced than that of *V. dahliae*.

The parasitism of *S. bataticola* is connected with a destruction of the living tissues of the host (root cortex), while *V. dahliae* lives in the xylem of the conducting system and has no destructive effect.

The different forms of nitrogen were determined in the usual way (²). The results are presented in Table 3.

It may be seen from the table that *Verticillium dahliae* contains nearly twice as much total and protein nitrogen as *Sclerotium bataticola*. Total nitrogen is increased mainly at the expense of protein nitrogen. The increase in the amount of protein nitrogen in *V. dahliae* is to be explained by the fact that the main mass of the fungus consisted of mycelium, while in *S. bataticola* it consisted of sclerotia.

The higher protein content of mycelium as compared to that of sclerotia is quite natural, for sclerotia are filled mainly with the reserve materials (carbohydrates and fats).

Attention is also attracted by the fact that a great amount of soluble nitrogen is present in both fungi; it differs but slightly from that of protein nitrogen. The soluble nitrogen contained in a large quantity in the two fungi was represented mainly by amine nitrogen, the amount of ammonia- and amide nitrogen found by us being insignificant (about 0.0175 mg of nitrogen pro 100 cm^3 of the extract).

In the medium after the cultivation of the two fungi there was found also a small amount of ammonia and of amides.

To demonstrate the toxic effect produced by the fungi upon the plants, experiments were arranged on the withering of newly cut branches of cotton which were kept in an aqueous extract prepared from the fungi and in a medium in which they have been cultivated during 2—3 days. The results are presented in Table 4.

Table 4

Influence of *Sclerotium bataticola* and *Verticillium dahliae* upon Withering of Plants

Liquids tested	Condition of branches of cotton plants	
	after 24 hrs.	after 48 hrs.
Water (control)	No changes. Leaves turgescent	No change. Leaves turgescent
Medium after cultivation of *S. bataticola*	Leaves lose turgor and wither; chlorotic spots appear on them	Branches withered. Leaves twist and dry up
Extract from *S. bataticola*	Do	Do
Medium after cultivation of *V. dahliae*	»	»
Extract from *V. dahliae*	»	»

A comparison of the data presented above shows that aqueous extract from either of the fungi, as well as the medium after their cultivation, cause a violent withering of the branches, which is manifest 24 hrs upon the beginning of the experiment; these liquids contain toxic substances.

The question as to the toxic effect produced by the parasite upon its host long since attracted the attention of many students ([1],[3], [5–7]). The writers disagree as to the interpretation of the nature of the toxines, yet the majority are inclined to believe the toxic substances to be the products of nitrogen exchange—ammonia, urea, amines, *etc.*

The analyses referred to above make us believe that the withering of the branches of cotton was caused not by ammonia or amides, whose amount was insignificant, but by some other products of nitrogen exchange.

From the study performed by us one may conclude that the galloping wilt, and probably also the trancheomicosis of cotton are connected with disturbances in the physiological processes of the plant and with its poisoning with toxic substances.

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EXPERIMENTAL MORPHOLOGY

ON THE DEGREE OF CORNEA DETERMINATION

By N. I. LASAREV

(Communicated by I. I. Schmalhausen, Member of the Academy, 10. XII. 1944)

The principal conclusion arrived at by all authors ([1–5]) engaged in the study of the morphogenesis of the cornea was that any part of the tadpole skin is able to transform into cornea if acted upon by the eye. On the other hand, if withheld from the necessary influence of the eye, the cornea of a tadpole loses its peculiarities, becomes turbid, pigmented and similar to the skin. These conclusions were extended to all frog species and to all the stages of tadpole development. The morphogenesis of the cornea is being cited as an example of «undetermined development», or as an examlpe of the necessity of «continuous induction» for the development of the cornea, *etc.*

In the course of current experimental work I required a pigmented cornea which I supposed to obtain by removing the eye from under the cornea in *Rana esculenta* tadpoles. The operation was carried out on 16 tadpoles. According to literature data, the pigmentation of the cornea begins on the 3—5th day, and is complete 15—25 days after the operation. The first examination of the tadpoles operated was accordingly fixed for the 15th day only. Contrary to anticipation, however, no traces of pigmentation could be observed. The cornea was completely transparent and distinctly delimited in all the tadpoles; in the subsequent the cornea in all the tadpoles that had survived remained clear and transparent during 45 days after operation. It should be mentioned that the operations were practised at the stage when two or three joints in the hind legs were present.

In order to check up these observations, similar operations were carried out on tadpoles of *Pelobates fuscus* at the same stage of development. The process of pigmentation was found to start 3—5 days after operation and to end 15 days after operation in all the operated tadpoles. Now I had every reason to believe that this time, unlike in all cases of reversible determination repeatedly observed before, I have succeeded in observing the phenomenon of a stable irreversible determination of the cornea. The phenomenon observed indicates in the first plane that there is no ground to consider the development of the cornea as being completely dependent upon the influence of inductor; neither is it possible to deny the self-differentiation capacity of the lens rudiment at the stage of the closing of the neural crests. In some amphibian species the lens is able to self-differentiate, while the cornea may be unpigmented. The fact that the two phenomena here discussed are especially clearly expressed in *R. esculenta* suggests the idea that both cornea pigmentation and cornea induction, like lens differentiation, proceed at a different rate in different amphibians. Besides, in some species this phenomenon may escape observation, thus giving rise to a cornea «row», similar to the lens «row» of Spemann. No definitive answer to this question can be given without special experiments. The perusal of the literature available went to confirm this suggestion.

The discovery of a stable determination of the cornea in *Rana esculenta* put forwards the same problem which led to the discovery of the phenomenon of lens self-differentiation in *Rana esculenta*. This problem is concerned with the cause of this frog species standing apart from the rest. It can be formulated as follows: whether this condition is due to the ability of the material to fix the condition acquired, or whether the compensatory influence of the system arisen upon the removal of the eye is operative here? From an analysis of this problem I came to the conclusion that, being very sensitive, the process of pigment formation may be prevented by the slightest influences exerted by the eye components remaining in the alveolum (muscles, nerves, *etc.*). To check this assumption it was sufficient to graft the tadpole cornea of the same stage on to the back which, of course, is free of any compensatory influence of the eye. If the cornea transplanted were also found stable under these conditions, one certainly would be obliged to refer the behaviour of the cornea of *Rana esculenta* to its peculiar ability to fix the inductive influences accumulated during the ontogenesis. An experiment carried out with the purpose to settle the question gave the following result. The cornea remained perfectly transparent after being kept on the back during 15 days after transplantation. However, during the next 5 days (15—20 days after operation) the transplanted cornea underwent an intense and rapid pigmentation. It could still be distinguished from the adjacent skin, its boundaries being sharply outlined, but the whole cornea transplanted was found to contain a lot of pigment granules. Consequently, the stability of cornea determination, or, better to say, the irreversibility of the cornea in *Rana esculenta* is due to the influence produced by the orbit. It is possible to suggest that the lens in *R. esculenta* is self-differentiating according to the same principle. However, there is some difference between the development of the cornea in *R. esculenta* and other species, since the pigmentation of the cornea in *Pelobates fuscus* and *Rana temporaria* begins 3—5 days after the operation, while in *R. esculenta* the pigment appears 15 days after operation.

Along with these experiments I also removed the eye from under the cornea in *Rana esculenta* at the stage when opercular perforation takes place. In this juncture I naturally paid attention to the rapid pigmentation of the cornea after operation. After some 4—5 days the «influx» of the pigment at the edges of the cornea could clearly be seen. There were only 6 tadpoles under observation, all of which showed a similar picture. Thus, in the development of *Rana esculenta* there is also a stage during which a development of the cornea pigment takes place.

The fact that the cornea is still transparent 40 days after the removal of the eye in tadpole is especially interesting. However, as soon as the process of metamorphosis begins, pigmentation of the cornea also begins, which shows that during metamorphosis the cornea of *R. esculenta* becomes dependent upon the eye influence again.

This fact seems to contradict to the current conception as to the degree of dependence of determination upon the age. It is generally adopted that the degree of determination increases with time until it becomes a stable irreversible determination. In the light of the experiments described above this contradiction can be given a plausible explanation. The process of pigment development was observed at the stage of metamorphosis. The latter, in turn, is connected with a decrease in the stability of the skin, as may be seen from its visible reorganisation (resorption of tail, appearance of opercular opening, and so on). On the basis of these considerations it is possible to suggest that the cornea as a «skin» element becomes less stable at the stage of metamorphosis. This suggestion is also strengthened by the fact that during metamorphosis the cornea undergoes reorganization (fusion of the internal and external corneas) and, therefore, becomes more sensitive to influences proceeding from the eye. This suggestion may, by the way, be considered as a plausible explanation of the fact that the eye is capable to induce both the lens and cornea during a very long period in ontogenesis.

On the basis of the results here described it is possible to conclude that the cornea in *Rana esculenta* tadpoles is firmly and irreversibly determined, and does

not develop any pigment after the eye had been removed from under it; it preserves at the same time its size, transparency, and clear outlines; the degree of the determination of the cornea in tadpoles varies according to species, age, and localization.

The main significance of the phenomenon observed of stable determination of the cornea in *R. esculenta* tadpoles consists in that it makes it evident that the process of cornea determination should not be cited as an example of an «undetermined» formation requiring a «continuous induction». The degree of determination of the cornea, as well as that of the lens determination, differs as to the species of amphibians. There exist both stable and unstable degrees of the cornea determination, and there is every reason to expect the existence of the «cornea row» similar to that of the «lens row» of Spemann. Being isolated from the eye, the lens preserves the shape of a lens, and develops no pigment because of its isolation.

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